MATH CURRICULUM

GRADE: SIX

HORACE W. PORTER SCHOOL, COLUMBIA, CONNECTICUT

| MANAGE W. LONIEN SCHOOL, COLUMNIC, COMMICCHICOL | ", COMMECTICOI |
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| GOALS/OBJECTIVES | RESOURCES/SUGGESTED ACTIVITIES |
| THE STUDENT WILL: | |
| MEASUREMENT | Middle School Mathematics Project |
| -Determine elapsed time. | -Mouse and Elephant: Measuring Growth III & IV |
| -Figure change for purchases. | The Math Solution Measurement Menu |
| -Measure lengths and draw lines to the nearest 1/8 inch | A Collection of Math Lessons, Gr. 6-8 |
| and nearest half centimeter. | |
| -Measure weight in kilograms, grams, ounces, and pounds. | |
| -Measure capacity in liters, milliliters, and customary | |
| units. | |
| -Measure to determine perimeter, areas, and volumes. | |
| -Identify appropriate customary or metric measures for a | |
| given situation. | |
| -Convert measures of length and time. | |
| | |
| GEOMETRY | Middle School Mathematics Project |
| -Identify, draw, describe, and classify geometric shapes and | -Spatial Visualization - Activity II |
| figures. | The Math Solution Geometry Menu |
| -Identify or draw points, lines, and segments. | Math & The Mind's Eye - Looking at Geometry |
| -Identify or draw geometric transformations and lines of | |
| symmetry. | |
| -Recognize when two figures appear to be congruent or | |
| similar. | |
| | |
| | |

CORE ACTIVITIES FOR GRADE 6 MATH EXPERIENCES

The following resources/activities should be experienced by all grade 6 students in order to facilitate consistent development of important math concepts and problem solving skills.

1. Addison-Wesley Problem Solving Supplement - Grade 6..

This is an instructional program consisting of 150 problem-solving experiences and a teaching strategy for problem solving. It may be taught via a problem-of-the-day approach, or as a complete unit, or in any other format the teacher desires. There are 30 clusters of five problems: a skill activity, a one-step problem, a multiple-step problem, and two process problems.

The critical instructional component is the student discussion that takes place after students have had an opportunity to work the problems. Allow for students to share their thinking and strategies for solving the problems. Teachers should focus on the processes/strategies used, rather than the answer, Also, encourage students to write frequently in math class.

The mathematics needed to solve the problems lags the curriculum objectives by about half a year. For example, the objectives needed to solve a problem in the middle of the third grade book were covered at the beginning of the third grade book.

- 2. Try-A-Tile for addition, subtraction, multiplication, and division practice with a problem solving focus.
- 3. The Palindrome Problem from <u>A Collection of Math Lessons</u>, Gr. 3-6. This is a good problem-solving activity for use with cooperative grouping. It also provides opportunities for computational practice and pattern searching.
- 4. The Popcorn Problem from <u>A Collection of Math Lessons</u>, Gr. 3-6. A range of mathematical concepts is used in this particular lesson. In the number area, students are asked to focus on ratio and proportion and fractional equivalence. In the statistics area, students collect, organize, represent, and interpret data, are introduced to frequency distribution, and explore mean, median and range. In the probability area, students discuss the representation of the probability of an event.
- 5. The Measurement Menu is a menu approach to a hands-on study of the metric system. Various activities require that student estimate, then find exact measurements. See <u>The Math Solution</u>.

6. The Middle School Mathematics Project:

Probability - Activities III and IV, pp 29-42 and 79-90 Spatial Visualization - Activity II, pp 36-48 Mouse and Elephant: Measuring Growth - Activities III and IV, pp 29-58 Similarity and Equivalent Fractions - Activities III and IV, pp 33-61 Factors and Multiples - Activities I and II, pp 1-33

- 7. The Geometry Menu is a menu approach to a hands-on study of geometry with a problem solving focus. See <u>The Math Solution</u>.
- 8. Algebra Activities What's My Rule, The Function Machine, What Comes Next, etc.
- 9. Various games and activities that promote development of number sense, logical thinking, etc.

Digit Place NDQ (No Dumb Questions) Guess My Number Riddles People Sorting Too High/Too Low Bicycle Problems Who Has? Creature Cards Base 10 Riddles **Guess My Rules** \$ Riddles w/Too High Too Low Scattered Numbers Rounding Riddles Arrow Math 20 Questions Match Game (Factors) Buzz Path Game Reject \$1.00 Words Mystery Stories Around the World

- 10. Get It Together is an excellent resource of activities for cooperative group problem solving.
- 11. OTHER RESOURCES

Moving On With Geoboards
Tangram Activities
Pentominoes
TOPS Deck - Level B
Calculator TOPS Deck
Addison-Wesley Mathematics Textbook - Grade 5

rev. 2/95

NCTM STANDARD 10: MEASUREMENT

In grades K-4,the mathematics curriculum should include measurement so that students can-

- . understand the attributes of length, capacity, weight, area, volume, time, temperature, and angle;
- develop the process of measuring and concepts related to units of measurement;
- . make and use estimates of measurement;
- . make and use measurements in problem and everyday situations

BY THE END OF GRADE 2, THE STUDENT WILL:

Outcome

- 1. Experience non-standard units of measure.
- 2. Have experiences with measurement vocabulary using the attributes of length, volume, weight, area, time, and temperature when describing their physical environment and their own bodies.
- 3. Estimate and measure length in meters, centimeters, yard, feet, inches
- 4. Estimate and measure weight and capacity using balance scales and large and small containers.
- 5. Use measurement in comparing, counting, estimating, and making one-to-one correspondence.
- 6. Tell time to the nearest hour, half-hour, and quarter hour.
- 7. Sequence days of the week and months of the year.
- 8. Participate in a variety of writing and speaking experiences using measurement vocabulary.
- 9. Solve a variety of problems involving measurement.
- 10. Begin to select an appropriate non-standard or standard unit of measure for length.

Sample Instructional Activities

- Measure items in room using clothespins, milk containers, paper clips, string, etc.
- Guessing Jar: Students take turns filling a jar with some object from home (m&m's, crackers, paper clips, etc.) and other students guess how many handfuls fit in jar, then how many objects. Students filling jar write in journal about their choice, how many, etc...
- Estimate how long it takes for some observable event to occur (ex: how long it takes a cup to sink to bottom of pail of water). Use non-standard measure: How many stars I can draw before...; How many times I can stand up, sit down before... How many circles can I draw in 1 minute?

Recommended Materials

unifix cubes, balance and weight scales, toothpicks, clothespins, paper clips, rulers, meter and yard sticks, containers for volume measure, clocks, thermometers

NCTM STANDARD 10: MEASUREMENT

In grades K-4, the mathematics curriculum should include measurement so that students can-

- . understand the attributes of length, capacity, weight, area, volume, time, temperature, and angle;
- . develop the process of measuring and concepts related to units of measurement:
- . make and use estimates of measurement;
- . make and use measurements in problem and everyday situations.

BY THE END OF GRADE 4, THE STUDENT WILL:

Outcome

- 1. Select, estimate, and use appropriate non-standard units of measure.
- 2. Make comparisons and estimate using:

volume in liters, ounces, pints quarts, gallons

weight in grams, kilograms, pounds ounces, tons

length in meters, centimeters, kilometers, yards, feet, inches, miles angles greater than, less than or equal to a right angle.

- 3. Measure volume, length, weight to the nearest 1/2 unit.
- 4. Select, estimate, and use appropriate units of measure within a system of linear measure.
- 5. Find perimeter of a regular or an irregular polygon.
- 6. Find the area of a rectangle and express it in square units.
- 7. Read and write and measure time in the following ways:
- a. to the nearest minute (digital and analog); to the month, day, and year
- b. describe elapsed time clock and calendar;
- c. know equivalents minutes/hour, hour/day, days/week, days/month, days/year

Sample Instructional Activities

- Reinforce use of measurement & selecting appropriate measurement (standard and non-standard) in other subjects: Art students design and lay out bulletin board Science students choose appropriate measures for an experiment; Social studies students make maps, with a scale, of some thing or place well known, and then compare to actual map & scale.
- Encourage measurement of objects and distances outside the classroom (playground, hallways, bus routes, etc...)
- Build into problem solving hidden information problems which require knowledge of measures and equivalents in order to solve.
- Use television scheduling and after school activities to determine elapsed time. Have students plan their own weekly schedules.
- Write a how-to letter to someone describing a measurement problem including a description of something like a bedroom. Use precise terms and accurate measurements and units.

- 8. Integrate manipulative usage and appropriate measurement vocabulary effectively communicate measurement information to others.
- 9. Solve problems involving measurement.
- 10. Begin to read and estimate and have a sense of temperature (Celsius & Fahrenheit) in relation to hot & cold.

Recommended Materials

double sided tape measures, rulers, meter sticks, yard sticks, balance scales and weights, containers for volume measure, thermometers, clocks, rolling meter measurer, yarn or string, Base 10 blocks, geoboards and elastics, maps.



GRADE 6

Strand—Estimation, Fractions, Decimals, Percents

Rational numbers describe the relationships of parts to a whole. Students must develop a variety of ways to think about, model, describe, and write about the relationships. Fractions, decimals, and percents are different notation systems for the same idea—ratio relationships. The concept of equivalence helps students understand rational numbers, select the most efficient notation to represent a rational number, and make good estimates of reasonable answers to problems.

Through cutting and comparing real objects, children develop an understanding of whole and parts which leads to the conceptual exploration of fractions that are part of the whole. By counting out equal subsets and comparing them, students explore ratio and the concept of parts of a set (i.e., 1/3 of 12 is 4 and 2/3 of 12 is 8). Patterns connect ratio concepts to multiplication and division.

Students engage in talking about the parts in relation to the whole and writing about them in different ways. They should not focus on the algorithms for manipulating the notation, but instead on the way fractions, decimals, and percents describe real objects or pictures of objects. Students need continuous experience using a variety of manipulative materials in order to develop the base of understanding that will support them as they develop ways to compute with fractions, decimals, and percent. Without these models, students mindlessly manipulate symbols, merge the steps in operations, or forget them. With the consistent use of models that support visualization, students are able to construct their own strategies, make more reasonable estimation, and verify their answers. Fractions, decimals, and percents have been combined into a single strand because they are natural and easier to teach in connection with one another.

The essential topics for instruction in fractions include naming fractions, comparing fractions, equivalence of fractions, and operations with fractions. It seems that fractions with numerators of 1 do not pose much of a problem for students. Yet when fractions such as 2/5, 3/4, and so on enter the picture, so does confusion. Also, children have difficulty deciding which of two fractions, 2/3 and 4/5, for example, or even 1/8 and 1/16, represents more, and why fractions such as 3/6 and 4/8 describe the same part of a unit. These types of difficulties indicate the need for further concept development with concrete materials and real-life situations.

When teaching students who lack basic understanding of fractions, it's not uncommon for teachers to feel the pressures of time and the demands of the curriculum. So, it's often a temptation to speed up instruction and to teach the rules for operating with fractions. Some teachers resort to telling students, for example, that multiplying the numerator and denominator by the same number gives an equivalent fraction, or that in order to divide you invert the fraction on the right and multiply across the tops and across the bottoms.



Working Towards CMT Objectives on the Grade 8 Mastery Test

- 2. Relate fractions, decimals, and percents to their pictorial representations.
- 3. Rename fractions and mixed numbers as equivalent decimals and vice ve-sa.
- 4. Rename fractions and decimals as equivalent percents and vice versa.
- 5. Identify points on number lines, scales, and grids including fractions, decimals, and integers.
- 6. Estimate the magnitude of mixed numbers and decimals.
- 7. Add and subtract 2-, 3- and 4-digit whole numbers, money amounts, and decimals.
- 8. Multiply and divide 2- and 3-digit whole numbers, money amounts and decimals by 1-digit whole numbers and decimals.
- 9. Multiply and divide whole numbers and decimals by 10, 100, and 1000.
- 10. Add and subtract fractions and mixed numbers with reasonable and appropriate denominators.
- 11. Multiply whole numbers and fractions by fractions and mixed numbers.
- 12. Find percents of whole numbers.
- 14. Identify an appropriate procedure for making estimates involving fraction and mixed number computation.
- 15. Identify an appropriate procedure for making estimates involving decimal computation.
- 16. Identify an appropriate procedure for making estimates involving percents.
- 17. Solve problems involving order and magnitude of fractions.
- 18. Solve problems involving order and magnitude of whole numbers and decimals.
- 19. Solve problems involving rounding whole numbers and decimals.
- 24. Solve or estimate a reasonable answer to problems involving fractions, decimals, and mixed numbers.
- 25. Solve or estimate a reasonable answer to problems involving ratios, proportions, and percents.



GRADE 6

Strand—Estimation, Fractions, Decimals, Percents

Giving students rules to help them develop facility with procedures will not help them understand the concepts. The risk is that when students forget a rule, they have no way to reason through a process. Try this assessment with your students: Give students a fraction problem they're "supposed" to understand, 1/2 + 1/3, for example. Ask them to show you what the problem means with any concrete material, or a drawing, or by relating it to some real-life situation. Listen to students' responses, and let their responses guide you when making instructional choices.

It is important to provide a variety of ways for students to learn about fractions—with concrete materials, from a geometric perspective, with a numerical focus, and related to real-life situations. Let the students know that different people learn in different ways, and that they should pay attention to the kinds of activities that help them develop understanding. Encourage them to try activities with which they are less comfortable. Do not expect immediate results from any one activity. Students need time to absorb new ideas and integrate them with the understanding they already have.

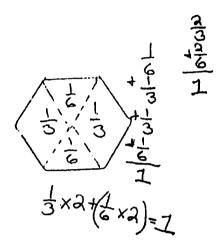
TEACHING

Activities **Objectives** Have students construct, use, describe, 6.40 Compare, order, and round draw and write about models and fractions and decimals. diagrams which show the value of (See also 6.44). fractions and provide a measurable model CMT Grade 8 Objectives 5, 6, 17, for comparison. For example, students 18.19. may model fractions by folding paper strips to represent unit fractions with denominators from 1 through 10 and compare the lengths. Students may use the number line as a diagram to locate and order fraction and decimal values.

Cluster—Rational Numbers Strand—Estimation, Fractions, Decimals, Percents

GRADE 6





A student's model of addition and multiplication with pattern blocks.

STRATEGIES Assessment **Materials and Resources** Ask students to demonstrate, describe, Use: 6.40 draw and write about models that will pattern blocks convince someone that: paper strips 1/2 = 6/12a) paper squares 2/3 > 2/4**b**) place value blocks 0.5 > 0.2c) rods and squares 3/5 is closer to 1/2 than 3/4 is. ten-strips and 10 x 10 grids tangrams See also Math in Stride Performance geoboards Assessment, Student Book 6, pages 26-27, number lines 62-64, 75, and 77. rulers See: Family Math, "Fractions Kit", 120-123. Math in Stride, Book 6, "Ordering Fractions and Decimals," Teacher

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Edition (TE) 112 and Student

GRADE 6



Strand—Estimation, Fractions, Decimals, Percents

| Materials and Resources | Assessment |
|---|--|
| Workbook (SW) 164-165. Math in Stride, Book 6, "Ordering Decimals" TE 117 and SW 170. | |
| pattern blocks place value blocks decimal squares fraction bars fraction circles See: Math in Stride, Book 6, "Equivalent Expressions for Fractions, Decimals, and Percents," TE 64- 66 and SW 94-103. Math in Stride, Book 6, "Simplifying Multiplication of Fractions with Prime Factors," TE 109-111 and SW 157-163. Math in Stride, Book 6, "Multiplication of Decimals as Multiplication of Fractions," TE 131-133 and SW 182-196. | Have students choose, draw, label and write about models that will convince someone that: a) 2/4 + 2/8 = 3/4 b) 1/2 + 1/3 ≠ 2/5 c) 1/2 + 1/10 = .50 + .10 d) Ask students to show at least three different ways to find 4 x 3-1/2. e) Ask students to draw a model that shows the answer to the problem: 1/4 x 3-1/2. |
| 6.42 Use: pattern blocks rulers paper folding place value blocks, strips and grids Explorer calculators | See the assessment tasks suggested in the Sample Lesson. See also Math in Stride Performance Assessment, Student Book 6, pages 7, 30, 50, and 75-79. |
| See: Family Math, "Judy's Fractions," page 125. Math in Stride, Book 6, | |

9, 11, 14, 15, 24.



Cluster—Rational Numbers

GRADE 6

Strand—Estimation, Fractions, Decimals, Percents

| Objectives | Activities |
|---|--|
| 6.41 Use multiple strategies to identify equivalent fractions and decimals, and to add, subtract, and multiply with them. (See also 6.44). CMT Grade 8 Objectives 2, 3, 4, 7 10, 11, 14, 15. | Extend the use of models and diagrams and the comparing and ordering of fractions and decimals to the identification of equivalent values, and estimation and computation. For example, use pattern blocks to prove that $1/3 = 2/6$ and that $1/2 + 1/3 = 5/6$. Use a 10×10 grid to show that $20/100 + 5/100 = 25/100$ (or $1/4$) and $.20 + .05 = .25$ (or $1/4$). |
| 6.42 Build and draw models to demonstrate the concepts of multiplication and division with fractions and decimals. (See also 6.44). CMT Grade 8 Objectives 2, 6, 8, | See the Clinton Sample Lesson— Connecting the Strands—An Investigation of Division with Fractions. This lesson helps students develop an understanding of multiplication and division with fractions and helps them connect the use |

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of patterns, estimation, and spatial models

of length and area. The lesson moves from conceptual understanding through models, diagrams, and real-world applications to working with fraction



GRADE 6

Strand—Estimation, Fractions, Decimals, Percents

| Objectives | | Activities | |
|------------|--|---|--|
| | | notation and patterns with the symbols, to using patterns with the Explorer calculator to find and use the reciprocal. | |
| 6.43 | Write and solve equations using mixed numbers, like and unlike fractions, and decimals that match word problems involving addition, subtraction, and multiplication. (See also 6.44). CMT Grade 8 Objectives 7, 8, 9, 10, 11, 14, 15, 17, 18, 19, 24. | Begin by having students write a story problem that could be solved using either of the following equations: 1/2 + 3/4 = x .50 + .75 = x Repeat with other equations. Then turn the process around and have students write and solve equations to solve practical problems, such as finding the amount in 3-1/4 boxes of candy bars if one dozen is in a box. | |
| 6.44 | Use, describe, and write about ways to estimate and identify reasonable answers to problems involving rational numbers. (See also 6.40-6.43). CMT Grade 8 Objectives 6, 14, 15, 17, 18, 19, 24. | Build an estimation step into the classroom discourse as students solve problems involving comparing, ordering, rounding, adding, subtracting, multiplying and solving story and practical word problems with fractions and decimals. Encourage students to predict reasonable answers and describe a variety of ways to | |

GRADE 6



Strand—Estimation, Fractions, Decimals, Percents

| Materials and Resources | Assessment |
|---|--|
| "Multiplication With a Model" and "Division With a Model," TE 59-62 and SW 88-91. Passport, Professional Handbook, Section H 92-H95. Passport, Student Text Lab 8.4, pages 370-371, and practice pages 372-375, and 377-385. Math Land, "Interpreting and Solving Problems Involving Multiplication and Division of Fractions," pages 126-133. | |
| 6.43 Use: Student selected models and diagrams, as needed. See: NCTM Standards, "Math as Problem Solving," pages 75-77, "Math as Communication," pages 78-80, and "Number and Number Relationships," pages 87-90. | See Math in Stride Performance Assessment, Student Book 6, pages 56-57 and 66-68. Also, have students create their own story problems and have other students try to solve them. |
| 6.44 See: Objectives 6.40-6.43. | Ask students to contrast two different ways to estimate an answer, and then use each way to do a different, but similar problem. For example, 0.12 x 3.52 could be about ≈ 1/10 x 3-1/2 → 1/10 x 3-5/10 or |



GRADE 6

Strand—Estimation, Fractions, Decimals, Percents

| Activities |
|---|
| estimate the answer to a problem. |
| Use the 10 x 10 grid drawn on centimeter graph paper and shade in some of the centimeter squares. For example, if 15 squares are shaded, you have 15 out of 100, or 15/100, or 0.15, or 15%. Shade in 25 squares and have students write as many symbols as they can to describe the diagram. For example, they might propose 25/100, 0.25, 1/4, and 25%. Experiment with other amounts of shaded centimeter squares. |
| Set up a table and look for patterns to help fill it in. For example: |
| |

Have students describe the patterns.

60, 30, or 20 objects.



GRADE 6

Strand—Estimation, Fractions, Decimals, Percents

| Objectives | Activities |
|------------|--|
| | Set up a data-gathering project and connect graphing the data onto a circle graph with estimating simple percents. For example, try the activities from the NCTM Addenda Series cited in the next column. |

GRADE 6



Strand—Estimation, Fractions, Decimals, Percents

| Materials and Resources | Assessment |
|--|--|
| | ≈ 0.10 x 3.5 → 0.35 Ask students to choose and defend what they think is the most efficient (easiest) method to estimate. |
| 6.45 Use: place value blocks 10 x 10 grids | See performance tasks in Math in Stride Performance Assessment, Student Book 6, pages 31, 56, 58, and 75-80. |
| See: CT Mastery Test Handbook—Second Generation—Math, the section on equivalence. Math in Stride, Book 6, "Fraction and Percent Equivalence," TE 114 and SW 167. | |
| 6.46 Use: grids and tables calculators computer spreadsheet/graphing/ packages with circle graphs. See: NCTM Addenda Series—K-6 Sixth | Have students gather some data and display it using the circle graph—approximating and sketching by hand. Then have students use a computer software package to construct and print a circle graph from the same data and write a description of the interpretation of each graph. |
| Grade Book, "Making Sense of Data," pages 16-18. NCTM Addenda Series—Grades 5-8—Understanding Rational Numbers and Proportions, "Investigation 3: Garbage and | See also performance tasks in Math in Stride Performance Assessment, Student Book 6, pages 56-57, 65, 80, and 107. |

GRADE 6



Strand—Estimation, Fractions, Decimals, Percents

| Materials and Resources | Assessment |
|------------------------------------|------------|
| Recycling: The Rational (Number) | |
| Approach," pages 32-48. | |
| Math in Stride Book 6, "Fractional | |
| Parts of a Percent," TE 134 and | |
| SW 197 and "Choosing Strategies | |
| to Solve Percent Problems," TE | |
| 135 and SW 198-199 and | |
| "Solving Percent Problems," TE | |
| 136 and SW 200-201. | |

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TRIGONOMETRY A #143

In Trigonometry, students apply a unified, holistic, problem-solving technique to measure indirectly and to project and design solution systems to represent real world periodic relationships.

The students will:

1. Perform basic operations with angles including measurement in degrees and radians, rotations, and co-terminal angles.

Concrete areas to which these concepts can be applied:

- . functions and relations,
- . distance formula,
- . arc length,
- . co-terminal solutions,
- . using scientific calculators and graphing calculator,
- . changing from degree measure to radian measure,
- . changing from radian measure to degree measure,
- . angular and linear velocity,
- . area of a sector.
- 2. Define and evaluate the trigonometric functions of an angle.

Concrete areas to which these concepts can be applied:

- . predicting values of trigonometric functions,
- . determining values using tables,
- . special cases (30-60-90) and (45-45-90),
- . using scientific and graphing calculator,
- . define functions using the unit circle.
- 3. Solve right triangles and related real world problems using trigonometric functions.

Concrete areas to which these concepts can be applied:

- . indirect measure using the trigonometric of a right triangle,
- . angles of elevation and depression,
- . word problem applications in navigation, surveying, and construction using line of sight and bearings,
- scientific calculators.

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4. Graph the trigonometric functions and their inverses on a Cartesian coordinate system.

Concrete areas to which these concepts can be applied:

- . amplitude, periods, phase shift, vertical shift, and symmetry,
- establishing identities and relationships,
- . rotations about axis and transformations,
- graphing calculator and computers,
- . predicting results,
- . composite functions.

5. Use fundamental trigonometric identities to write equivalent trigonometric expressions and prove other identities.

Concrete areas to which these concepts can be applied:

- . substitution.
- . sum and difference of two angle measures identifies,
- . double angle identities,
- . half angle identities,
- . product and sum identities,
- . graphing calculator and computers to verify and analyze results,
- . solving trigonometric equations.

6. Use the laws of sines, cosines, and tangents to solve triangles and find their area.

Concrete areas to which these concepts can be applied:

- . oblique triangles,
- . ambiguous case of triangles,
- . Heron's formula,
- . word problem applications in navigation, surveying, and construction using line of sight and bearings,
- . scientific calculators.

Define and convert polar coordinates and vectors.

Concrete areas to which these concepts can be applied:

- . graphing calculators,
- . operations of complex numbers in polar form,
- . converting rectangular coordinates to polar coordinates,
- . converting polar coordinates to rectangular coordinates,
- . developing and predicting graphs,
- . parametric functions,
- . applications of vectors.

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8. (optional) Define the use of harmonic motion sequence and series in application and solutions of patterned projections.

Concrete areas to which these concepts can be applied:

- . composite functions,
- . graphing calculators and computers,
- . predicting results,
- . establishing identities and relationships.

TWO SAMPLE CRITERION-REFERENCED ASSESSMENTS

1. November Mathematics Assessment Test – Grade 3, Pages 207-218.

From the Simsbury, Conn. Public Schools Used with permission from: Rosanne Daniele, Frances Solomon and Nancy Robbins.

2. Basic Algebra I Final Exam, Pages 219-229.

From Nonnewaug High School, Woodbury, Conn. Used with permission from James Paniati.

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| NOVEMBER | MATHEMATICS | ASSESSMENT | TEST | - | GRADE | 3 |
|----------|-------------|------------|------|---|-------|---|
|----------|-------------|------------|------|---|-------|---|

STUDENT PROFILE

| NAME | TEACHER | | · · · · · · · · · · · · · · · · · · · | |
|------------|---|-------|---------------------------------------|---|
| CONC | CEPTS | Items | Total | Number Wrong |
| 1. | Extended patterns for numbers and attributes. | 1-3 | 3 | |
| 2. | Identify alternative forms of expressing whole numbers using expanded notation. | 4-6 | 3 | |
| 3. | Relate addition and subtraction facts to pictures. | 7-8 | 2 | |
| 4. | Identify alternative forms of expressing whole numbers using regrouping. | 9-10 | 2 | *************************************** |
| 5. | Rounding to the nearest 10. | 11-13 | 3 | |
| 6. | Identify >, <, =. | 14-16 | 3 | |
| | Total | | 16 | |
| <u>NUM</u> | BER FACTS AND COMPUTATION | | | |
| 7. | Addition and subtraction fact families. | 17-19 | 3 | |
| 8. | Identify the missing addend. | 20-21 | 2 | • |
| 9. | Add and subtract facts to 18. | 22-25 | 4 | |
| 10. | Add and subtract 1- and 2-digit numbers without regrouping. | 26-29 | 4 | |
| | Total | | 13 | |

| PROF | BLEM SOLVING/APPLICATIONS | Items | Total | Number Wrong |
|------|--|-------|-------|-----------------|
| | Solve problems involving order and magnitude of whole number. | 30-31 | 2 | |
| 12. | Solve problems involving place value concepts such as 1 more/less, 10 more/less. | 32-33 | 2 | |
| 13. | Write story problems from number sentences. | 34-35 | 2 | |
| 14. | Identify the appropriate operation (addition or subtraction) to solve story. | 36-37 | 2 | |
| 15. | Solve simple story problems involving addition or subtraction. | 38-39 | 2 | |
| 16. | Solve problems rounding 2-digit whole number. | 40-41 | 2 | |
| 17. | Identify needed information in problem. | 42-43 | 2 | |
| | Total | | 14 | |
| ME? | ASUREMENT/GEOMETRY | | | |
| 18 | Tell time to the nearest hour, half hour. | 44-46 | 3 | |
| 19 | . Determine the value of a set of coins. | 47-48 | 2 | |
| 20 | . Estimate lengths and areas. | 49-50 | 2 | |
| | Total | | 7 | |
| | GRAND TOTAL | | 50 | |

SIMSBURY PUBLIC SCHOOLS GRADE3 NOVEMBER ASSESSMENT
NOVEMBER MATHEMATICS ASSESSMENT TEST - GRADE 3

1. What letter comes next in the pattern?

B C B D B E B F B

2. Fill in the missing number.

| 10 | 14 | 18 |
|----|----|----|
| 14 | | 22 |
| 18 | 22 | 26 |

3. What shapes come next in the pattern?



4. Which means the same as 300 + 80 + 7?

- a. 30,807
- b. 300,807
- c. 3087
- d. 387

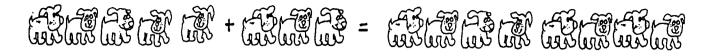
5. Which means the same as 523?

- a. 500 + 200 + 30
- b. 500 + 20 + 3
- c. 50 + 20 + 3
 - d. 500 + 20 + 30

6. Fill in the blanks.

67 = ____ tens + ____ ones

7.



8.



- 9. Circle the correct answer.
 - 61 means the same as:
 - a. 5 tens and 11 ones
 - b. 5 tens and 1 one
 - c. 1 ten and 16 ones
 - d. 4 tens and 11 ones
- 10. Which means the same as:
 - 7 tens and 11 ones
 - a. 81
 - b. 117
 - c. 71
 - d. 711

Round these numbers to the nearest 10.

- 11. 24 ---->
- 12. 37 ---->
- 13. 45 ---->

Put the correct sign in the circles.

- 14. 27 () 27
- 15. 19 () 27
- 16. 27 () 13

Complete each fact family.

- 17. 3 + 4 = 7
 - 4 + 3 = 7
 - 7 4 = 3

- 18. 6 + 2 = 8
 - 8 2 = 6
 - 2 + 6 = 8

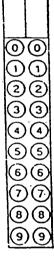
19. Use these numbers to write a fact family: 5, 4, 9

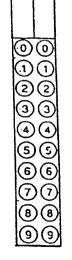
- a. = ____
- b. = ____
- c. =
- d. _____ = ____
- 20. + 1 = 10 21. 11 -
- 21. 11 = 5

Write your answer.

Write your answer and shade in the grid for the following problems:

| @@ (1)(1) | |
|--------------|--|
| 22 33 | |
| 4 | |
| 66 | |
| 88 | |







| 30. | Jane has 93ϕ , Pete has 57ϕ , and Mary has 64ϕ . Order the amounts of |
|-----|--|
| | money they have from the smallest to the largest. |
| | |
| | |
| 31. | Jim's teacher asked him to put these numbers in order from the greatest |
| | to the smallest. The numbers are 243, 316, and 271. How should Jim |
| | have answered? |

Write the answer in the grid and shade in the bubbles for the following problems:

32. Marcie had 70¢ in her pocket. Her mom gave her 10¢ more. Now how much money does Marcie have?

| (O) | |
|------------------------|--|
| 22 33 | |
| (4) (3) (3) | |
| 66 77 | |
| 8 9 9 | |

33. What number is 10 less than 50?

| 00 |
|------------|
| \bigcirc |
| 22 |
| 33 |
| 4 |
| 33 |
| 66 |
| 7 |
| 88 |
| 1 |
| 99 |

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| Write a s | tory prob | olem for | this | number | sentend | ce: 7 | - | 3 = | · |
| Write a s | | | | number | sentend | ce: 7 | - | 3 = | = |
| | | | | number | sentend | ce: 7 | - | 3 = | |
| | | | | number | sentend | ce: 7 | - | 3 = | |
| | | | | number | sentend | ce: 7 | - | 3 = | |
| | | | | number | sentend | ce: 7 | - | 3 = | |
| | | | | number | sentend | ce: 7 | | 3 = | |
| | | | | number | sentend | ce: 7 | - | 3 = | |
| | | | | number | sentend | ce: 7 | - | 3 = | |
| | | | | number | sentend | ce: 7 | | 3 = | |
| | | | | number | sentend | ce: 7 | | 3 = | |
| | | | | number | sentend | ce: 7 | - | 3 = | |

circle the expression that fits.

- 36. Lisa had 12 apples. She used 8 of them to make applesauce. How many were left?
 - 12 + 8 a.
 - b. 8 - 12
 - c. 12 8
 - d. 8 + 12

- 37. Jim had 6 stamps. He bought 7 more How many stamps does he have altogether?
 - a. 7 6
 - b. 13 7
 - c. 6 + 7
 - d. 13 6

Solve the problems.

38. Bob Caught 7 fish. Ed caught 9. How many did they catch altogether?

39. There are 9 children at one table. Four went to the teacher's desk. How many were left at the table?

Circle the correct answer.

- There are 57 football cards in Don's collection. About how many cards does he have? Round this number to the nearest 10.
 - a. 55
- b. 50
- c. 60 d. 65
- 41. Sue sold 22 bracelets. About how many bracelets is this?
 - a. a little less than 20
 - b. a little more than 20
 - c. a little less than 30
 - d. a little more than 30

Is there enough information to solve these problems? If not, tell what you need to know.

- 42. Tom and Mary each had a bag of marbles. Tom had 8 marbles. How many did they have in all?
- 43. Juan has some hockey cards. He gave three away. How many does he have left?

Write the time.

44.



45.



46.



Circle the correct answer.

47. Connie bought her sister a sticker book. Here is what Connie gave the clerk in the store.



a. 64¢

b. \$1.00

c. 54¢

d. 52¢

48. Donna had 57¢ to buy some fruit. Which set of coins could she have?









| 40 | Thout | how | manv | squares | will | cover | the | rectangle? |
|-----|-------|-----|------|---------|---------|-------|------|------------|
| 49. | ADOUL | поw | many | adrarea | M T T T | COACT | 0110 | 10000 |

a. 2

b. 6

c. 8

đ. 4

50. About how many small squares will fit in the large square?

a. 4

b. 6

c. 9

d. 12



BASIC ALGEBRA I Final Exam June 1995

| 1. | Define Function. |
|----|---|
| 2. | State the three ways to represent a function. |
| 3. | Give an example of the following mathematical ideas from the situation "I am studying for a test" |
| | a. variable- b. constant- c. input variable- d. output variable- e. function- |
| 4. | State the order of operations. |
| | |
| | |
| 5. | For the following situation sketch a reasonable graph. In the space provided, write a paragraph that clearly and fully explains your graph. |
| | SITUATION: When food is cooked and then set out on the table, its temperature depends on time. |
| | *Label axes. |
| | 1. Label 2. Graph |
| | |
| | |
| | 3. Explanation |
| | |
| | |
| | |
| | |
| De | veloped by James Paniati, Nonnewaug High School, Woodbury, Connecticut |
| | |

6. At the Strikes to Spare bowling lanes the management offers a special deal to attract large groups on Sunday. There is a \$25 charge for the group and then \$0.50 per game bowled. That means that for the group the total cost of an outing at Strikes and Spare depends on the number of games bowled. If n represents the total number of games bowled, the rule for the total cost is: C(n) = .5n + 25.

Answer each question below. Use any method you wish.

- a. How much will it cost if a group bowls 24 games? SHOW WORK.
- b. How many games gave been bowled if the total cost is \$52? SHOW WORK.
- c. Is this a linear function? Explain how you know.

In 7-9 simplify the given expression.

- 7. 4(3x 6)
- 8. 7n + 13 2n + 5
- 9. 3x + 6(3x 2)

In 10-11 multiply the binomials.

- 10. (x + 4) (x + 3)
- 11. (3x 6)(x + 2)

12. Situation - Owners of a movie theater chain are studying business prospects for a new location in Hyattsville. They believe that daily profit of the theater will be a function of ticket price. The pattern of that relation is given by the following table. Use this table to answer a-e which follow.

| Ticket Price t in dollars | Profit P(t) in dollars |
|---------------------------|------------------------|
| 0.00 | -400 |
| 0.50 | -260 |
| 1.00 | -150 |
| 1.50 | -65 |
| 2.00 | 0 |
| 2.50 | 50 |
| 3.00 | 90 |
| 3.50 | 120 |
| 4.00 | 140 |
| 4.50 | 150 |
| 5.00 | 155 |
| 5.50 | 150 |
| 6.00 | 140 |
| 6.50 | 120 |
| 7.00 | 90 |
| 7.50 | 50 |
| 8.00 | 0 |

| a. | According | to | the | table, | setting | the | ticket | price | at | \$3.50 | will | result | in | a | profit |
|----|-----------|----|-----|--------|---------|-----|--------|-------|----|--------|------|--------|----|---|--------|
| | of | | | | | | | | | | | | | | |

- b. According to the table, which ticket price(s) should result in a daily profit of \$50.00?
- c. What information about the business situation is given by "P(1.50) = -65"?
- d. According to the table, which change in ticket price would cause greater change in daily profit? (Check the one that will.)

| Increase from S | \$1.50 to \$2.00 | or | Increase from | \$2.50 to | \$3.00 |
|-----------------|----------------------|----|---------------|-----------|--------|
| | 4 - 10 4 - 10 | | | • | |

e. Complete the following sentence to describe the overall relation between ticket price and daily profit given in the table:

| As | ticket | price | increases, | profit |
|----|--------|-------|------------|--------|
| | | | | |

In problems 13-19 solve the given equation analytically (mechanically). If the equation is a quadratic use the quadratic formula. Show all steps for each problem.

13.
$$6x = 22.5$$

$$14. \ 5x + 7 = 42$$

15.
$$\frac{3}{5}x - 12 = 11.4$$

16.
$$7y + 3 = y + 9$$

Quadratic Formula

$$x = -b + b2 - 4ac$$

$$2a$$

$$17. -5x2 - 4x + 1 = 0$$

$$x = -b - b2 - 4ac$$
2a

18.
$$3x^2 + 5x + 7 = 0$$

19.
$$4x2 - 20x + 30 = 5$$

In questions 20-22 solve the following equations graphically using the TI-22. State the solution and indicate the window.

20. 8.3 x - 17 = 7.9

State the window values of your graph

x-min _____ x-max ____

y-min ____

y-max ____

Solution x =

21. 6x - 5 = 37

x-min ____

x-max ____

y-min ____

y-max ____

Solution x =

22. -5x2 - 18x + 16 = 23

Make a quick sketch of the TI-82 graph you viewed to solve the equation.

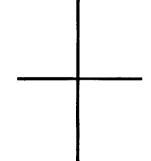
x-min ____

x-max ____

y-min _____

y-max ____

x = ____

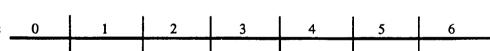


- 23. The U R Fit Health Spa offers two different membership packages. The first deal requires a membership fee of \$100 plus \$25.50 a month. The second deal requires a membership fee of \$140 plus \$18.50 a month.
 - a. Complete the following table of values for each of the situations described above.

Package #1
of months
Total Cost of
Membership

| 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| | | | | | | |

Package #2 # of months Total Cost of Membership



- Graph each of the above functions on the same set of axes. Be sure to label your axes and clearly indicate the scale. Use the graph paper provided.
- c. If x = the number of months and y = the total cost, state the rule that represents each membership package.

Rule for package #1

Rule for package #2

d. Are these membership packages represented by linear functions? Explain clearly how you decided.

Answer the questions e-i based on the table, the graph or the rule that represent the Health Spa situation.

e. How many months did you belong if your total membership cost was at least \$325?

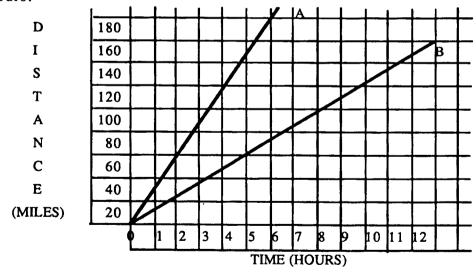
| Package | #1 |
|---------|----|
| Package | #2 |

f. What is the rate of change for each of these membership packages? (2 answers)

| Package | #1 | per |
|---------|----|-----|
| Package | #2 | per |

- g. When is the package #2 a cheaper deal?
- h. Write an explanation of why some might choose package #1.
- i Which is a better deal? Explain your answer.

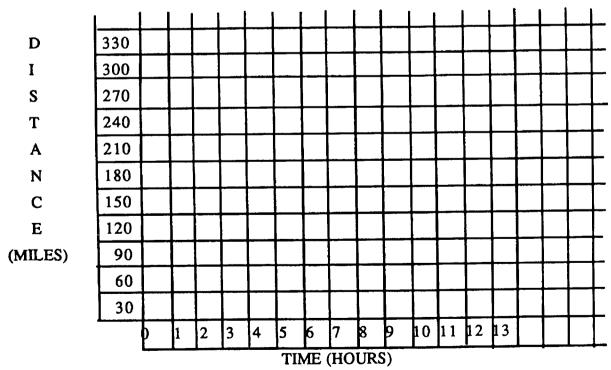
24. Below is a graph comparing distance traveled as time passed for two different cars.



a. For car A use the coordinates of the indicated points to determine the speed of the car in miles per hour.

SHOW WORK

- b. For car B you need to select two points you wish to use and then calculate the speed. (All units are already in miles per hour.) SHOW WORK
- c. After 20 hours how far has each car gone? Two answers, one for each ca
- d. How long does it take each car to travel 1200 miles? Two answers, one fo each car.
- 25. Draw the graph of each of the following cars below. Do all graphs on the coordinate plane below.
 - a. Car C left home driving 45 miles per hour.
 - b. Car D started 300 miles from home and headed toward home at 30 miles per hour.



26. Graph using data table method. Show all work.

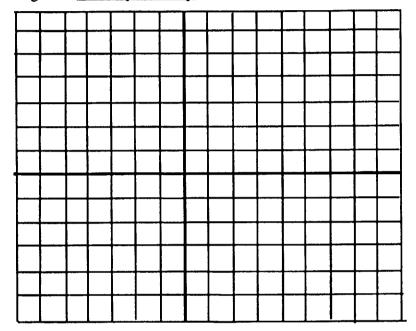
$$y = x + 7$$

| x | у |
|---|---|
| | |
| | |

27. Graph each by finding the x and y intercepts. Show all work.

$$3x - 4y = 24$$

*Graph #26 and #27 on the graph to the right.

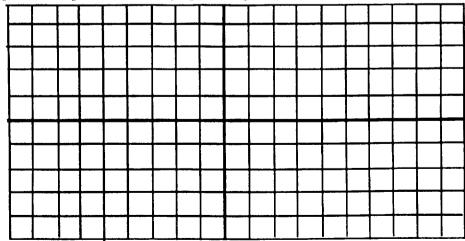


28 Graph by using the slope intercept method.

$$y = \frac{3}{5} x - 6$$

29. Change to slope intercept form and graph using any method.

$$6x + 2y = 14$$



30. For the following points, find a linear rule to fit the data. Show all work. Make sure to check the rule.

Formulas needed.

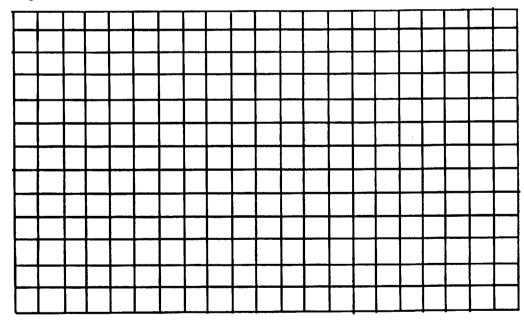
$$\begin{array}{c|cccc}
x & y \\
\hline
2 & 6 \\
5 & 18
\end{array}$$

$$\begin{array}{ccccc}
m & = \underbrace{y2 - y1}_{x2 - x1} \\
y - y1 & = m(x - x1)
\end{array}$$

31. The data below show a relation between monthly sales of a soft drink and money spent on television advertising.

Money spent on Advertising (in thousands of \$) 100 125 175 250 Sales 500 600 725 825 (in thousands of \$)

a. Graph the data below. Label axes. Put numbers along each axes.



b. Find the best rule for the data. Show all work. Formulas needed.

$$m = y2 - y1$$

 $x2 - x1$ $y - y1 = m(x - x1)$

c. State the slope of your line and explain what it means in this problem.

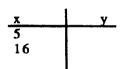
- d. Based on your rule predict the effect on sales of \$300 thousand in advertising.
- e. Based on your rule, how much money did you spend on advertising if you made \$900 thousand in sales?
- f. Give a reason why the data is not perfectly linear.
- 32. Give an example of each.
 - a. linear monomial

- b. quadratic trinomial
- c. 4th degree polynomial with 5 terms
- 33. Given 5x4 the 5 is called the _____ the 4 is called the _____
- 34. For each of the following solve for y.

Fill in data table,

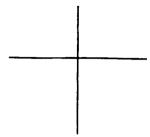
Show all work.

a.
$$y = 2x^2 + 3x - 5$$



- b. $y = 4x^2 + 3x$ Find y when x = -2
- 35. For the following chart of data, decide if it represents a linear or a quadratic function. Explain how you decided.
 - x -2 -1 0 1 2 y 17 9 3 -1 -3

36. On the graph below sketch an example of a quadratic equation with 2 real solutions.



37. Ann Archer shoots an arrow into the air with an initial upward velocity of 50 meters per second.

Use Vertical Motion Formula: d = -5t2 + rt

- a. How high is the arrow after 4 seconds?

 Solve this problem analytically and not graphically.

 SHOW ALL STEPS.
- b. When will it be 80 meters high? (Remember to find two answers)

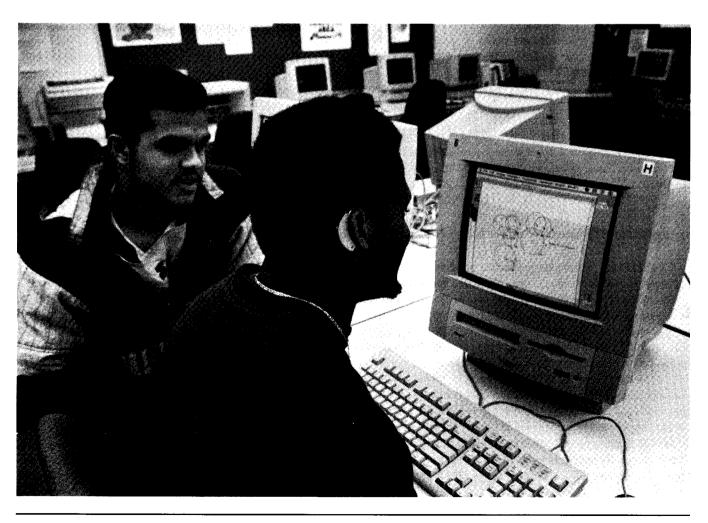
 Use Quadratic Formula: $x = -b + \sqrt{b^2 4ac}$ $x = -b \sqrt{b^2 4ac}$ 2a
- 38. Chuck throws a rock with an initial velocity of 30 meters per second.

 Solve the following with either the TI-82 or analytically. You do not need to show any work.
 - a. What is the altitude after 2 seconds?
 - b. When is it 25 meters above from where it was thrown?
 - c. When is it back down at the throwers level?

| CHAPTER | Δ |
|---------|---|
| CHAPIER | |

Critical Issues In Mathematics Programs

Reading And Writing In Mathematics
Integrating Mathematics Across Disciplines
Customary And Metric Measurement
Grouping, Tracking And Inclusion
Manipulative Materials
Calculators And Computers
Alternative Scheduling
Equity And Diversity
Parent Involvement And Support
Recognizing A Good Math Classroom



As educators use the content delineated in Chapter 2, the program components described in Chapter 3, and the development process outlined in Chapter 4, they will face several additional issues that tend to be unique to mathematics. Each of these issues is complex and comes with few easy answers. Each must be faced head-on if we are to truly make mathematics work for all.

Reading And Writing In Mathematics

It used to be that the "three R's" were separate and distinct. It was thought that a lot of mathematics – meaning arithmetic – could be learned regardless of a student's reading and writing abilities. However, as applications and problem solving have become more significant parts of the mathematics program, and as the language arts have been more broadly integrated across the curriculum, reading and writing have assumed an increasing role in high-quality mathematics programs.

Reading skills become more and more important as teachers reduce the emphasis on pages of wordless, rote computation and symbol manipulation and increase the emphasis on problems, applications, and the use of contexts like those illustrated in Chapter 2. Similarly, writing skills become more and more important as teachers use projects, open-ended tasks and other written means to develop, as well as assess, mathematical understanding and communication.

In fact, the shift toward making mathematics instruction as language-rich as possible reinforces language arts skills as well as improves the quality of the mathematics program. Rather than making it more difficult to learn mathematics, language-rich programs have been shown to be more effective for all students, particularly for those with limited English proficiency.

Integrating Mathematics Across Disciplines

Just as the mathematics program is a necessary and natural place to reinforce language arts skills, the science, social studies, health education, arts, home economics and technology education programs are ideal places into which mathematics can be integrated, and from which rich and relevant contexts for applying mathematical skills and concepts can be drawn.

Mathematics, therefore, should have a visible place in the curriculums of other disciplines and other disciplines should be clearly visible within the contexts, tasks, projects and activities of the mathematics program. Whether linking art to geometry, statistics to a science experiment, or technology education to measurement, these cross-disciplinary connections must be carefully

built into the curriculum development process and can significantly enhance the relevance of learning for students.

Customary And Metric Measurement

The ongoing debate over when or whether the United States will "go metric" leaves the mathematics curriculum developer in the somewhat awkward position of having to do both. Any observer of the current state of affairs and the world of the foreseeable future will recognize that our students must be prepared to be "bilingual" in terms of measurement. That is, the world will continue to bombard Americans with quarts of milk, twoacre lots, two-by-fours for studs and tons of trash - all requiring a thorough familiarity with the customary system of length, weight and capacity. However, Americans are increasingly confronted with 25-milligram doses, 2-liter bottles and 1500-meter races that all argue for an equally thorough familiarity with the metric system of length, mass and volume. Like the Connecticut Mastery Test (CMT), the Connecticut Academic Performance Test (CAPT), and most recently produced instructional materials, an effective program provides a balanced approach to measurement units - one part customary and one part metric.

Perhaps more important than the issue of customary vs. metric, however, is the broader issue of the importance of measurement generally. The study of measurement - customary and metric - is frequently omitted from the implemented curriculum, and performance on some CMT and CAPT items reveals serious gaps in student understanding of basic measurement. Therefore, the key issue is assuring adequate attention to the concepts of measurement through the use of nonstandard measures (e.g., pencil lengths or paper clip weights), and equally strong attention to both systems of measurement units. In fact, a curriculum based on problem solving is one that regularly asks questions like the following: How far? How much? How big? How much bigger? These questions and others like them all have answers with units like minutes, dollars, ounces, pounds, milligrams or meters, as well as square feet and kilometers per hour. These kinds of problems should encourage all to increase the curricular emphasis on and the instructional time allotted to measurement.

Grouping, Tracking And Inclusion

What was written in Chapter 3 bears repeating: "No single component of the educational system more powerfully communicates the expectations – both high and

low – we hold for young people than the ways in which schools group, sort and track students. A major step in moving toward the vision of "mathematics for ALL" is a dramatic decrease in the ability-grouping and tracking of students. This does not mean abandoning all ability groups; it does not mean the elimination of all honors courses; and it does not mean all students grouped heterogeneously all of the time. It does mean, however, a change in policy at the school and district levels regarding ability-grouping and tracking so that no student is denied access to a rich and demanding mathematics program best suited to his or her individual needs and interests."

The grouping and tracking of students was an acceptable and inevitable strategy for implementing a skill-based mathematics program, assessed with norm-referenced tests and designed primarily to sort out students for a world of work that was based on many assembly line workers and few managers. As technology has diminished the need for mastery of complex paper-and-pencil skills, and as the world of work has demanded broader problem-solving skills on the part of nearly all students, the rationale for tracking and grouping has started to weaken. The increasing call for inclusion and greater heterogeneity must be seen as a call for greater opportunity for all. It is also a call for changes in instructional patterns so that students are much more actively involved in their learning.

Regardless of grouping policies, most teachers are well aware of the broad range of abilities, styles, interests, motivations and rates of learning in every class, and these teachers recognize that they must provide for the individual needs of all students, be they high, low or in-between. The fact that more heterogeneous classes, often including special education students, usually are more difficult to teach must be balanced with the understanding that research shows clearly that tracking:

- increases the gap of opportunity to learn;
- results in unfair and disproportionate placement of poor and minority students in lower tracks;
- inappropriately isolates students primarily on the basis of how fast they learn;
- results in lower track students never catching up; and
- lowers self-esteem and increases behavior problems of lower-track students.

These outcomes, even if the result of the best of intentions, are no longer compatible with the needs of today's workplace or the goals of our democracy.

Manipulative Materials

Manipulatives – concrete physical objects that can be viewed, handled, arranged and taken apart to illustrate and exemplify concepts – are highly useful tools in mathematics classrooms at all levels. Research demonstrates clearly that most people learn better with hands-on experiences. Manipulatives are powerful tools for these hands-on experiences and serve as a natural bridge from concrete to abstract understanding of mathematical ideas. In addition, these materials appeal to a variety of senses and, thereby, provide greater access to mathematical ideas.

As is the case with all tools, the use of manipulatives is not nearly as important as **how** they are used. Such critical understandings as the concept of number, the meaning of place value, the concept of fraction, the "why" of the division algorithm, the meaning of "mean," or visualizing what happens when an area is rotated into a volume in calculus, all can be more effectively taught through the use of manipulative materials. It is important to remember that students cannot learn mathematics well with manipulatives alone. However, instruction that makes effective use of manipulatives is likely to be far more effective than instruction based solely on text-books and chalkboards. (Appendix E provides crosswalks between core concepts and key manipulative materials.)

Calculators And Computers

What the pencil was to yesterday's mathematics and writing, the calculator and the word processor are to today's mathematics and writing: critical tools that, when used appropriately, enhance learning.

The heart of the debate over the appropriate use of technology in mathematics classes is whether these tools are labor-saving crutches that discourage the development of key skills and discipline or powerful instructional tools that strengthen problem-solving skills, provide increased access to mathematical ideas and prepare students for the world of work, where technology is ubiquitous. The underlying philosophy of this guide, as well as that of the National Council of Teachers of Mathematics, is the latter – that calculators and computers are essential tools for teaching mathematics and should permeate a high-quality program. Obviously, as with all tools, there is a time and a place for their use. An effective program recognizes that, while students are learning to master basic number facts and to develop estimation skills, calculator use should be very limited.

While the power of calculators is constantly changing, it is recommended that primary students have access, when appropriate, to simple four-function calculators to solve problems, develop place value understandings, and strengthen their command of the four basic operations. Intermediate students should have access to fraction-capable calculators that, in addition to what fourfunction devices can do, allow students to strengthen fraction and decimal number sense. From middle school through high school it is recommended that some form of graphics calculator with a multiline screen, graphing and tabular representations, and all the features of a scientific calculator be available to strengthen prealgebraic, algebraic and statistical understanding. In fact, the illustrative tasks described in Chapter 2 assume the foregoing calculator recommendations.

As calculators become more and more powerful, the line between what is a calculator and what is a computer becomes more and more blurred. For example, many are calling the TI-92, with a QWERTY keyboard, a "hand-held computer," since it is capable of symbol manipulation, extensive data manipulation and dynamical geometry. However, until such hand-held computers are on every student's desk, the classroom computer can be used for demonstration and for independent work.

Alternative Scheduling

At the time of this writing, another controversial issue – particularly at the high school level – is alternative scheduling schemes, including the block schedule. For many, block scheduling is an idea whose time has come, and changing the core schedule by which high schools are organized is a long overdue step toward revitalizing these schools. From the perspective of the mathematics program, proponents argue that block schedules offer several powerful improvements for teachers and students.

First, it is widely recognized that a hands-on, activity-based laboratory approach, complete with projects, tasks, experiments and technology, requires more time than the traditional 45- or 50-minute class period. Block schedules that provide 80- and 90-minute periods are more conducive to the kinds of instructional shifts being advocated. Second, it is widely recognized that a daily load of 100 to 140 students is much too large if one is going to assign additional tasks and provide meaningful feedback on student work. Schedules that reduce student course loads to three or four courses per semester also reduce teacher loads to three courses and approximately 75 students, creating more reasonable

opportunities for larger-scale projects and longer-term activities.

A major drawback that teachers of mathematics face within such block schedules is the possibility that students may go nearly a year without a mathematics course. Proponents of block scheduling say that this is rare, due to the fact that students are taking more mathematics and most students follow a consistent pattern of mathematics each first semester or each second semester. Others suggest that there is little difference between a semester of geometry in a block schedule and a full year of geometry in a traditional schedule, and that concerns about the need for daily exposure to mathematics are unfounded.

Equity And Diversity

Equity and diversity are broad goals of every school district. However, like all broad goals, they must be translated into daily classroom practice. Such equitable classrooms are easy to recognize.

- All students, girls as well as boys, African-American and Hispanic as well as white, shy as well as assertive, quiet as well as loud, speak up in class regularly.
- No student is allowed to "put down" or pick on another student.
- High expectations are communicated clearly to all students, not just those who appear to "get it."
- Praise is based on achievement, not neatness of work, compliance or appearance.
- All students experience leadership roles.
- All students have an opportunity to learn the material presented.
- The contributions of female and male mathematicians and scientists from different ethnic groups are embedded within the curriculum.
- Stereotypes about who does and who does not do mathematics are confronted directly.

These characteristics are key pieces of a program that truly makes mathematics work for all and, therefore, must be components of a professional development program, as well as of the professional supervision and evaluation processes. A more detailed "Profile of an Equitable Math and Science Classroom and Teacher" can be found in Appendix G.

Parent Involvement And Support

It is increasingly clear that no real reform of mathematics programs is likely until there is parental support for the changes being proposed. Parents are legitimately wary that reform is just a new version of the "new mathematics" of the past. They are legitimately concerned that educators once again are experimenting with their children. And they are legitimately worried that this reform will be just another passing fad that leaves their children unable to compete in the workplace.

These concerns can only be addressed through ongoing communication and the development of trusted relationships between parents and schools. This communication can be fostered by inviting parents into classrooms, encouraging parents to volunteer during mathematics classes, and being available to answer parental questions and concerns. This communication also occurs through annual "math nights" conducted at schools,

periodic letters home explaining the mathematics program in which students will be engaged, and conferences that allow parents to express their concerns and view the results of the program.

In addition, parental support can be garnered by encouraging parents to make mathematics commonplace at home. Parents can help their children and support the mathematics program by making the mathematics of everyday life a part of the interaction with their children. This can be done in fast-food restaurants and with menus when students estimate costs and tips; at the gas pump where decimals and rates are clearly displayed; at the bank with the deposits and withdrawals; with cooking recipes which include fractions and measures; at sports events and with the newspaper sports section filled with statistics; or at the grocery store with costs and unit costs. This "conversational math" that helps build number sense and estimation can be as important as reading to children.

10 Expectations Parents Should Have About Their Children's K-8 Mathematics Program

In forging a positive relationship with parents, it is important that parents have high and clear expectations for their children's achievement. The following "10 Expectations Parents Should Have About Their Children's K-8 Mathematics Program" were developed by the Connecticut Academy for Education in Mathematics, Science and Technology.

- 1. Parents should expect that the way mathematics is being taught to their children will be very different from the way they were taught 20 or 30 years ago.
- 2. Parents should expect that their children will be using calculators regularly.
- Parents should expect to see their children doing few repetitive and tedious drills, such as multiplication tables and long division.
- Parents should expect that their children's mathematics classes and homework will include solving interesting and relevant mathematics problems, gathering and analyzing data, justifying solutions and writing conclusions.
- Parents should expect that the mathematics their children are learning will be ben-

- eficial and applicable to life outside of school.
- Parents should expect that their children will be prepared and encouraged to take algebra and geometry during their high school years.
- 7. Parents should expect that their children's mathematics achievement will be assessed and reported on the basis of their problemsolving abilities, projects and portfolios of work (done individually or as part of a group), not on the basis of mastery tests and standardized tests alone.
- Parents should expect that mathematics will be enjoyable for their children and that mathematics classes, activities and assignments include hands-on experiences that are likely to excite and encourage their youngsters.
- 9. Parents should expect that, if or when these expectations are not being met, they (and their questions) will be welcomed by school personnel; and that parents will be valued for caring enough to ask questions.
- 10. Finally, parents should expect that learning takes work and discipline.

Recognizing A Good Math Classroom

The Mathematical Sciences Education Board, in 1997, developed two sets of indicators – what both students and teachers would be doing – in order to recognize "a good math classroom when I see it." These indicators, published in a brochure *What Should I Look For in a Math Classroom?* are reprinted with permission.

A math classroom should provide practical experience in mathematical skills that are a bridge to the real world of jobs and adult responsibilities. This means going beyond memorization into a world of reasoning and problem solving. Sounds good, but how will I recognize a good math classroom when I see it?

What Are Students Doing?

- ☐ Interacting with each other, as well as working independently, just as adults do.
- Using textbooks as one of many resources. Students should know how and when to use manipulatives (such as blocks and scales) and technology (such as calculators and computers) as problem-solving tools.
- Applying math to real-life problems and not just practicing a collection of isolated skills. Lots of time is allowed for solving complex problems.
- Seeking a best solution among several solutions to a problem. Students can explain the different ways they reach these solutions, and defend the choice of one over another.
- Working in groups to test solutions to problems with each group member highly involved.
- Communicating mathematical ideas to one another through examples, demonstrations, models, drawings and logical arguments.
- Working in teams to challenge and defend possible solutions. Students help each other to learn.

What Are Teachers Doing?

- Guiding students in exploring multiple solutions to any problem; challenging students to think deeply.
- Moving around the room to keep everyone engaged in productive work.
- Encouraging students to raise and discuss questions about math for which there are no textbook answers. Rather than simply answering these questions, teachers are helping students to gain mathematical competence and confidence by finding their own solutions.
- Guiding students in making appropriate use of manipulatives and technology.
- Promoting student use of inquiry and creativity. Students are moved to higher levels of learning by pursuing alternative approaches to solving a problem or by proposing new problems that are variations on, or extensions of, a given problem.
- Bringing a variety of learning resources, including guest presenters, into the classroom in order to increase learning options for all students.
- Working with other teachers to make connections between disciplines to show how math is a part of other major subjects that students are studying.
- Using assessment that focuses on problem solving and understanding rather than on memory and speed.
- Helping all students to explore career opportunities that use the mathematics that they are learning.

From What Should I Look For in a Math Classroom?

Adapted from Creating a Climate for Change . . . Math Leads The Way.

Produced by the Math Connection and funded by the Annenberg/
CPB Math and Science Project, Washington, DC, 1998. Used with
permission. Bulk copies of this in brochure form can
be ordered (for a minimal fee) by calling 1-800-965-7373.

-SCANS -

WHAT WORK REQUIRES OF SCHOOLS * * * * * A SCANS REPORT FOR AMERICA 2000

THE SECRETARY'S COMMISSION ON ACHIEVING NECESSARY SKILLS U.S. DEPARTMENT OF LABOR



FIGURE B

FIVE COMPETENCIES

Resources: Identifies, organizes, plans, and allocates resources

- A. *Time*—Selects goal-relevant activities, ranks them, allocates time, and prepares and follows schedules
- B. Money Uses or prepares budgets, makes forecasts, keeps records, and makes adjustments to meet objectives
- C. Material and Facilities Acquires, stores, allocates, and uses materials or space efficiently
- D. Human Resources Assesses skills and distributes work accordingly, evaluates performance and provides feedback

Interpersonal: Works with others

- A. Participates as Member of a Team—contributes to group effort
- B. Teaches Others New Skills
- C. Serves Clients/Customers works to satisfy customers' expectations
- D. Exercises Leadership—communicates ideas to justify position, persuades and convinces others, responsibly challenges existing procedures and policies
- E. Negotiates works toward agreements involving exchange of resources, resolves divergent interests
- F. Works with Diversity works well with men and women from diverse backgrounds

Information: Acquires and uses information

- A. Acquires and Evaluates Information
- B. Organizes and Maintains Information
- C. Interprets and Communicates Information
- D. Uses Computers to Process Information

Systems: Understands complex inter-relationships

- A. *Understands Systems*—knows how social, organizational, and technological systems work and operates effectively with them
- B. Monitors and Corrects Performance—distinguishes trends, predicts impacts on system operations, diagnoses deviations in systems' performance and corrects malfunctions
- C. Improves or Designs Systems suggests modifications to existing systems and develops new or alternative systems to improve performance

Technology: Works with a variety of technologies

- A. Selects Technology—chooses procedures, tools or equipment including computers and related technologies
- B. Applies Technology to Task—Understands overall intent and proper procedures for setup and operation of equipment
- C. Maintains and Troubleshoots Equipment Prevents, identifies, or solves problems with equipment, including computers and other technologies

FIGURE C

A THREE-PART FOUNDATION

Basic Skills: Reads, writes, performs arithmetic and mathematical operations, listens, and speaks

- A. Reading—locates, understands, and interprets written information in prose and in documents such as manuals, graphs, and schedules
- B. Writing—communicates thoughts, ideas, information, and messages in writing; and creates documents such as letters, directions, manuals, reports, graphs, and flow charts
- C. Arithmetic/Mathematics performs basic computations and approaches practical problems by choosing appropriately from a variety of mathematical techniques
- D. Listening—receives, attends to, interprets, and responds to verbal messages and other cues
- E. Speaking—organizes ideas and communicates orally

Thinking Skills: Thinks creatively, makes decisions, solves problems, visualizes, knows how to learn and reasons

- A. Creative Thinking—generates new ideas
- B. *Decision Making*—specifies goals and constraints, generates alternatives, considers risks, and evaluates and chooses best alternative
- C. Problem Solving—recognizes problems and devises and implements plan of action
- D. Seeing Things in the Mind's Eye—organizes, and processes symbols, pictures, graphs, objects and other information
- E. Knowing How to Learn—uses efficient learning techniques to acquire and apply new knowledge and skills
- F. Reasoning—discovers a rule or principle underlying the relationship between two or more objects and applies it in solving a problem

Personal Qualities: Displays responsibility, self-esteem, sociability, self-management, and integrity and honesty

- A. Responsibility exerts a high level of effort and perseveres towards goal attainment
- B. Self-Esteem believes in own self-worth and maintains a positive view of self
- C. Sociability—demonstrates understanding, friendliness, adaptability, empathy, and politeness in group settings
- D. Self-Management—assesses self accurately, sets personal goals, monitors progress, and exhibits self-control
- E. Integrity/Honesty chooses ethical courses of action

CONNECTICUT MASTERY TEST THIRD GENERATION BLUEPRINT

Grades 4, 6 and 8 Content Overview Effective Fall 2000

| Content Standard | Strand |
|----------------------------|--|
| Number Sense | 1. Place Value |
| | 2. Pictorial Representations of Numbers |
| | 3. Equivalent Fractions, Decimals and |
| | Percents |
| | 4. Order, Magnitude and Rounding |
| Operations | 5. Models for Operations |
| | 6. Basic Facts |
| | 7. Compute with Whole Numbers and |
| | Decimals |
| | 8. Compute with Fractions |
| | 9. Solve Word Problems |
| Estimation and | 10. Numerical Estimation Strategies |
| Approximation | 11. Estimate Solutions to Problems |
| Ratios, Proportions | 12. Ratios and Proportions |
| and Percents | 13. Compute with Percents |
| Measurement | 14. Time |
| | 15. Approximate/Estimate Customary |
| | and Metric Measures |
| | 16. Customary and Metric Measures |
| Spatial Relationships | 17. Geometric Shapes and Properties |
| and Geometry | 18. Spatial Relationships |
| | |
| Probability and Statistics | 19. Tables, Graphs and Charts |
| | 20. Statistics and Data Analysis |
| | 21. Probability |
| Patterns | 22. Patterns |
| Alaches and Eunstians | 23. Algebraic Concepts |
| Algebra and Functions | 25. Migeorate Concepts |
| Discrete Mathematics | 24. Classification and Logical Reasoning |
| Integrated Understandings | 25. Mathematical Applications |
| 0 | • • |

GRADE 4

| Content Standard | Strand | Concept or Skill |
|------------------|---|--|
| Number Sense | Place Value | 1a. Solve problems involving 1 and 10, more or less. |
| | | 1b. Identify alternative forms of expressing whole numbers using expanded notation. |
| | | 1c. Identify alternative forms of expressing whole numbers using regrouping. |
| | | 1d. Use place value concepts to interpret the meaning of numbers. |
| | Pictorial Representations of Numbers | 2a. Relate pictorial representations using base ten blocks to whole numbers and vice versa. |
| | | 2b. Identify, label and shade fractional parts of regions and sets using pictures. |
| | Equivalent Fractions, Decimals and Percents | Not tested at Grade 4. |
| | Order, Magnitude and Rounding | 4a. Order whole numbers. |
| | and Rounding | 4b. Describe magnitude of whole numbers. |
| | | 4c. Round whole numbers. |
| | | 4d. Identify points representing whole numbers on a number line and vice versa. |
| Operations | Models for Operation | 5a. Relate multiplication and division facts to rectangular arrays and pictures. |
| | | 5b. Identify or write the appropriate operation or number sentence to solve a story problem. |
| | | 5c. Write story problems from addition and subtraction number sentences. |
| | Basic Facts | 6a. Add and subtract facts to 18. |
| | | 6b. Multiply and divide by 2,5 and 10. |

GRADE 4, continued

| Content Standard | Strand | Concept or Skill |
|------------------------------|--|---|
| | Compute with Whole Numbers and Decimals | 7a. Add and subtract 1- and 2-digit numbers without regrouping. |
| | | 7b. Add 1- and 2-digit numbers with regrouping. |
| | Compute with Fractions | Not tested at Grade 4. |
| | Solve Word Problems | 9a. Solve simple story problems involving addition or subtraction. |
| | | 9b. Solve simple story problems involving addition or subtraction with extraneous information. |
| Estimation and Approximation | Numerical Estimation Strategies | 10a. Identify the best expression to find an estimate. |
| | | 10b. Identify whether and why a particular strategy will result in an overestimate or an underestimate. |
| | | 10c. Determine the reasonableness or unreasonableness of an estimate and explain why. |
| | Estimate Solutions to Problems | 11a. Estimate a reasonable answer to a problem. |
| | | 11b. Use estimation to make and defend decisions. |
| Ratios, Proportions | Ratios and Proportions | Not tested at Grade 4. |
| and Percents | Compute with Percents | Not tested at Grade 4. |
| Measurement | Time | 14a. Tell time to the nearest hour, half-hour and quarter-hour using analog and digital clocks. |
| | | 14b. Solve problems involving time, elapsed time and calendars. |
| | | 14c. Interpret situations involving clocks and calendars. |
| | Approximate/Estimate Customary and Metric Measures | 15. Estimate lengths and areas. |

GRADE 4, continued

| Content Standard | Strand | Concept or Skill |
|------------------------------------|---|--|
| | Customary and Metric Measures | 16a. Measure or draw lengths to the nearest inch or centimeter. |
| | | 16b. Identify appropriate customary or metric measure for a given situation. |
| Spatial Relationships and Geometry | Geometric Shapes and Properties | 17a. Identify geometric shapes and figures, including the number of angles and sides of polygons. |
| | | 17b. Draw geometric shapes and figures. |
| | Spatial Relationships | Not tested at Grade 4. |
| Probability and Statistics | Tables, Graphs and Charts | 19a. Identify correct information from graphs, tables and charts. |
| | | 19b. Create bar graphs and pictographs from data in tables and charts. |
| | Statistics and Data Analysis | Not tested at Grade 4. |
| | Probability | 21. Solve problems involving elementary notions of probability. |
| Patterns | Patterns | 22. Extend or complete patterns involving whole numbers and attributes and identify or state rules for given patterns. |
| | | Not tested at Grade 4. |
| Algebra and Functions | Algebraic Concepts | 24a. Identify objects that are the same or dif- |
| Discrete Mathematics | Classification and Logical Reasoning | ferent by one attribute. |
| | | 24b. Sort objects into two groups by a common attribute. |
| Integrated Understandings | Mathematical Applications | 25. Solve extended numerical and statistical problems. |

GRADE 6

| Content Standard | Strand | Concept or Skill |
|------------------|---|---|
| Number Sense | Place Value | 1a. Solve problems involving 100 and 1,000, more or less. |
| | | 1b. Identify alternative forms of expressing whole numbers <10,000 using expanded notation. |
| | | 1c. Identify alternative forms of expressing whole numbers <10,000 using regrouping. |
| | | 1d. Use place value concepts to interpret the meaning of numbers. |
| | Pictorial Representations of Numbers | 2a. Relate decimals (0.01 - 2.99) to pictorial representations and vice versa. |
| | | 2b. Relate fractions and mixed numbers to pictures and vice versa. |
| | | 2c. Construct pictorial representations of fractions, mixed numbers and decimals. |
| | Equivalent Fractions, Decimals and Percents | 3a. Rename equivalent fractions. |
| | | 3b. Rename equivalent mixed numbers and improper fractions. |
| | Order, Magnitude and Rounding | 4a. Order whole numbers less than 100,000. |
| | | 4b. Order fractions, mixed numbers and decimals. |
| | | 4c. Describe magnitude of whole numbers <100,000. |
| | | 4d. Describe magnitude of fractions, mixed numbers and decimals. |
| | | 4e. Round whole numbers. |
| | | 4f. Round decimals. |
| | | 4g. Locate points on number lines and scales. |
| Operations | Models for Operations | 5a. Identify the appropriate operation or number sentence to solve a story problem. |
| | | 5b. Write story problems from multiplication and division number sentences. |
| | Basic Facts | 6. Multiply and divide facts. |

GRADE 6, continued

| Content Standard | Strand | Concept or Skill |
|------------------------------|--|---|
| | Compute with Whole Numbers and Decimals | 7a. Add and subtract 2-, 3- and 4-digit whole numbers and money amounts less than \$100. |
| | | 7b. Multiply and divide multiples of 10 and 100 by 10 and 100. |
| | | 7c. Multiply and divide 2- and 3-digit whole numbers and money amounts less than \$10 by 1-digit numbers. |
| | Compute with Fractions | 8. Add and subtract fractions and mixed numbers with like denominators. |
| | Solve Word Problems | 9a. Solve 1-step problems involving whole numbers and money amounts. |
| | | 9b. Solve 2-step problems involving whole numbers and money amounts. |
| | | 9c. Solve 2-step problems and explain how the solution was arrived at. |
| Estimation and Approximation | Numerical Estimation Strategies | 10a. Identify the best expression to find an estimate. |
| | | 10b. Identify whether and why a particular strategy will result in an overestimate or an underestimate. |
| | | 10c. Determine the reasonableness or unreasonableness of an answer or estimate. |
| | Estimate Solutions to Problems | 11a. Estimate a reasonable answer to a problem and explain why it is reasonable. |
| | | 11b. Use estimation to make and defend decisions. |
| Ratios, Proportions | Ratios and Proportions | Not tested at Grade 6. |
| and Percents | Compute with Percents | Not tested at Grade 6. |
| Measurement | Time | 14a. Solve problems involving elapsed time. |
| | | 14b. Solve problems involving the conversion of measures of time. |
| | Approximate/Estimate Customary and Metric Measures | 15. Estimate lengths and areas. |

GRADE 6, continued

| Content Standard | Strand | Concept or Skill |
|------------------------------------|---|--|
| | Customary and Metric Measures | 16a. Solve problems involving the conversion of measures of length. |
| | | 16b. Measure lengths to the metric or custom- ary unit specified. |
| | | 16c. Measure/determine perimeter and area. |
| | | 16d. Identify appropriate customary or metric units of measure for a given situation (length, capacity, mass). |
| Spatial Relationships and Geometry | Geometric Shapes and Properties | 17a. Identify, draw, describe and classify geometric shapes and figures. |
| | | 17b. Describe and classify geometric shapes and figures. |
| | Spatial Relationships | 18a. Identify or draw lines of symmetry. |
| | | 18b. Identify congruent figures. |
| | | 18c. Locate points on grids. |
| Probability and Statistics | Tables, Graphs and Charts | 19a. Identify correct information from graphs, tables and charts. |
| | | 19b. Create bar graphs and pictographs from data in tables and charts. |
| | Statistics and Data Analysis | 20. Draw and justify reasonable conclusions from graphs, tables and charts. |
| | Probability | 21. Solve problems involving elementary notions of probability and fairness, including justifying answers. |
| Patterns | Patterns | 22. Extend or complete patterns involving numbers and attributes and identify or state rules for given patterns. |
| Algebra and Functions | Algebraic Concepts | 23. Solve simple 1-step equations. |
| Discrete Mathematics | Classification and Logical Reasoning | 24. Solve problems involving the organization of data. |
| Integrated Understandings | Mathematical Applications | 25. Solve extended numerical, spatial and statistical problems. |

GRADE 8

| Content Standard | Strand | Concept or Skill |
|------------------|---|--|
| Number Sense | Place Value | 1a. Solve problems involving 0.1 and 0.01, more or less. |
| | | 1b. Identify alternative forms of expressing numbers using expanded notation. |
| | | 1c. Identify alternative forms of expressing numbers using scientific notation. |
| | Pictorial Representations of Numbers | Relate fractions, decimals and percents to their pictorial representations and vice versa. |
| | | 2b. Construct pictorial representations of fractions, decimals and percents. |
| | Equivalent Fractions, Decimals and Percents | 3a. Rename equivalent fractions and mixed numbers as equivalent decimals and vice versa. |
| | | 3b. Rename fractions and decimals as equiva- lent percents and vice versa. |
| | Order, Magnitude | 4a. Order whole numbers or decimals. |
| | and Rounding | 4b. Order fractions and mixed numbers. |
| | | 4c. Describe magnitude of whole numbers and decimals. |
| | | 4d. Describe magnitude of fractions and mixed numbers. |
| | | 4e. Round whole numbers, fractions and decimals. |
| | | 4f. Locate points on number lines and scales, including fractions, decimals and integers. |
| Operations | Models for Operations | 5a. Identify the appropriate operation or number sentence to solve a story prob- lem. |
| | | 5b. Write story problems from equations. |
| | Basic Facts | Not tested at Grade 8. |

GRADE 8, continued

| Content Standard | Strand | Concept or Skill |
|------------------------------|--|--|
| | Compute with Whole Numbers and Decimals | 7a. Add and subtract 2-, 3- and 4-digit whole numbers and decimals. |
| | | 7b. Multiply and divide whole numbers and decimals by 10, 100 and 1,000. |
| | | 7c. Multiply and divide 2- and 3-digit whole numbers and money amounts and decimals by 1-digit numbers and decimals. |
| | Compute with Fractions | 8a. Add and subtract fractions and mixed numbers with reasonable and appropriate denominators. |
| | | 8b. Multiply whole numbers and fractions by fractions and mixed numbers. |
| | Solve Word Problems | 9a. Solve 1-step problems involving whole numbers, decimals and money amounts. |
| | | 9b. Solve 1-step problems involving fractions and mixed numbers. |
| | | 9c. Solve multi-step problems involving whole numbers, decimals, fractions and mixed numbers, including averaging. |
| | | 9d. Solve problems involving whole numbers, decimals, fractions and mixed numbers with extraneous information. |
| | | 9e. Solve multistep problems and explain how the solution was arrived at. |
| Estimation and Approximation | Numerical Estimation Strategies | 10a. Identify the best expression to find an es timate. |
| | | 10b. Identify whether and why a particula strategy will result in an overestimate o an underestimate. |
| | | 10c. Determine the reasonableness or unrea sonableness of an answer or estimate. |
| | Estimate Solutions to Problems | 11a. Estimate a reasonable answer to a problem. |
| | | 11b. Use estimation to make and defend a decision. |

GRADE 8, continued

| Content Standard | Strand | Concept or Skill |
|------------------------------------|--|---|
| Ratios, Proportions and Percents | Ratios and Proportions | 12a. Solve problems involving ratios. |
| and I citches | | 12b. Solve problems involving proportions. |
| | Compute with Percents | 13a. Find percents of whole numbers or the percent a given number is of another number. |
| | | 13b. Solve problems involving percents. |
| Measurement | Time | Not tested at Grade 8. |
| | Approximate/Estimate Customary and Metric Measures | 15. Estimate lengths, areas and angle measures. |
| | Customary and Metric Measures | 16a. Solve problems involving the conversion of units of measure, including time. |
| | | 16b. Measure/determine perimeter, area and volume. |
| | | 16c. Identify appropriate customary or metric units of measure for a given situation. |
| Spatial Relationships and Geometry | Geometric Shapes and Properties | 17a. Identify and draw geometric shapes and figures. |
| | | 17b. Describe and classify geometric shapes and figures. |
| | Spatial Relationships | 18a. Identify or draw geometric transformations. |
| | | 18b. Identify, draw and describe lines of symmetry. |
| | | 18c. Relate 2- and 3-dimensional representations. |
| | | 18d. Identify and describe congruent and similar figures. |
| | | 18e. Locate points on grids. |
| Probability and Statistics | Tables, Graphs and Charts | 19a. Identify correct information from graphs, tables and charts. |
| | | 19b. Create graphs from data in tables and charts. |

GRADE 8, continued

| Content Standard | Strand | Concept or Skill |
|------------------------------|---|--|
| | Statistics and Data Analysis | 20a. Draw and justify reasonable conclusions from graphs, tables and charts. |
| | | 20b. Solve problems involving means and medians of sets of data. |
| | Probability | 21a. Solve problems involving elementary notions of probability and fairness, including justifying answers. |
| | | 21b. Solve problems involving expected outcomes or predictions. |
| Patterns | Patterns | 22. Extend or complete patterns involving numbers and attributes and identify or state rules for given patterns. |
| Algebra and Functions | Algebraic Concepts | 23a. Solve simple 1-step equations. |
| | | 23b. Use order of operations. |
| | | 23c. Evaluate expressions and use formulas. |
| | | 23d. Represent situations with algebraic expressions. |
| Discrete Mathematics | Classification and Logical Reasoning | 24. Solve problems involving the organization of data. |
| Integrated Understandings | Mathematical Applications | 25. Solve extended numerical, spatial statistical problems. |

CONNECTICUT ACADEMIC PERFORMANCE TEST (CAPT) SECOND GENERATION BLUEPRINT Content Framework Effective Spring 2001

All CAPT mathematics items will be in a context and require students to **solve a problem**. In addition, all openended CAPT items will require students to show their work and explain their **reasoning**, thereby **communicating** their understanding of the relevant mathematics.

All CAPT items will be devised to measure one or more of the following aspects of mathematical content:

The Number and Quantity Strand

1. Number Sense

- Use integers, fractions, decimals, percents and scientific notation in real-world situations to count, measure, compare, order, scale, locate and label.
- Use a variety of representations (including graphs, tables, words, number lines, pictures, etc.) to present, interpret and communicate various kinds of numerical information.
- Demonstrate an understanding of order, magnitude and equivalent forms of numbers.

2. Operations

- Identify appropriate operations (including addition, subtraction, multiplication, division, exponentiation and square roots) and use these operations in a variety of contexts.
- Select and use appropriate methods for computing (including mental mathematics, paper-and-pencil, and calculator methods).

3. Estimation and Approximation

- Select and use estimation strategies in problem situations.
- Assess the reasonableness of answers to problems.

4. Ratios, Proportions and Percents

- Use ratios, proportions and percents to solve problems.
- Use dimensional analysis to determine equivalent rates (for example, converting inches per minute to feet per hour).
- Use direct and inverse variation to solve numerical, geometric and algebraic problems.

The Measurement and Geometry Strand

5. Measurement

- Use the concepts of length, perimeter, area, volume, angle measure, capacity, weight and mass to solve problems, using both metric and customary units.
- Identify appropriate metric and customary measurement units and use appropriate measurement tools (including rulers and protractors).
- Estimate, make and use measurements in realistic situations.
- Use formulas and scales to determine measures.

6. Spatial Relationships and Geometry

- Interpret, describe and draw 2- and 3-dimensional objects.
- Use the concepts of rotation, reflection and translation to transform geometric figures.
- Describe and use fundamental concepts and properties of, and relationships among, points, lines, planes, angles and shapes (including incidence, parallelism, perpendicularity, and the Pythagorean Theorem).
- Use the concepts of congruence and similarity to solve realistic problems.
- Use coordinate representations of geometric figures.
- Solve problems using geometric models.

The Statistics, Probability and Discrete Mathematics Strand

7. Probability and Statistics

- Demonstrate an understanding of sampling and its role in statistical assertions.
- Describe, calculate and apply the concepts of mean, median, mode and range.
- Construct, read and interpret tables, charts and graphs of real-world data.
- Make and evaluate inferences from tables, charts, graphs and other representations of data.
- Use probability to make predictions and evaluate the likelihood of simple and compound events.
- Use simulations to determine experimental probabilities.
- Compare experimental and theoretical probabilities and make predictions based on these probabilities.

8. Discrete Mathematics

- Use systematic listing and counting strategies, including simple combinations and permutations, to solve problems.
- Use recursive processes, including iteration, to solve problems.

The Algebra and Functions Strand

9. Patterns

- Construct, describe, extend, and analyze a variety of numerical, geometric, and statistical patterns.
- Describe, analyze and generalize patterns using tables, rules, algebraic expressions and equations, and graphs.
- Make and justify predictions based on patterns.

10. Algebra and Functions

- Represent and analyze situations involving variable quantities with tables, graphs, verbal rules and equations, and translate among representations.
- Use variables, expressions, equations and inequalities, including formulas, to model situations and solve problems.
- Construct and use linear functions to model and solve real-world situations.
- Use the coordinate plane to represent functions.

Scoring Rubric for Mathematics Open-Ended Items

Each score category contains a range of student responses which reflect the descriptions given below.

Score 3

The student has demonstrated a full and complete understanding of all concepts and processes embodied in this application. The student has addressed the task in a mathematically sound manner. The response contains evidence of the student's competence in problem-solving and reasoning, computing and estimating, and communicating to the full extent that these processes apply to the specified task. The response may, however, contain minor arithmetic errors that do not detract from a demonstration of full understanding.

Score 2

The student has demonstrated a reasonable understanding of the essential mathematical concepts and processes embodied in this application. The student's response contains most of the attributes of an appropriate response including a mathematically sound approach and evidence of competence with applicable mathematical processes, but contains flaws that do not diminish countervailing evidence that the student comprehends the essential mathematical ideas addressed by this task. Such flaws include errors ascribable to faulty reading, writing, or drawing skills; errors ascribable to insufficient, non-mathematical knowledge; and errors ascribable to negligent or inattentive execution of mathematical processes or algorithms.

Score 1

The student has demonstrated a limited understanding of some of the concepts and process embodied in this application. The student's response contains some of the attributes of an appropriate response, but lacks convincing evidence that the student fully comprehends the essential mathematical ideas addressed by this task. Such deficits include evidence of insufficient mathematical knowledge; errors in fundamental mathematical procedures; and other omissions or anomalies that bring into question the extent of the student's ability to solve problems of this general type.

Score 0

The student has demonstrated merely an acquaintance with the topic. The student's response is associated with the task in the item, but contains few attributes of an appropriate response. There are significant omissions or anomalies that indicate a basic lack of comprehension in regard to the mathematical ideas and procedures necessary to adequately address the specified task. No evidence is present to suggest that the student has the ability to solve problems of this general type.

CAPT CALCULATOR USE POLICY

In setting this policy governing the use of calculators on the CAPT, the State Department of Education was guided by the following facts and beliefs:

Facts

- Some CAPT mathematics items are unreasonable to do without at least a four-function calculator.
- All CAPT items can be completed with a four-function calculator.
- The additional time students may gain if they use a graphics calculator is offset by the fact that the State Department of Education will provide enough time for all students regardless of the calculator they use to reasonably complete the test.

Beliefs

- Fairness is not necessarily achieved when everyone uses the same calculator, but rather when students are allowed to use the calculator with which they are most familiar and comfortable.
- Students are most likely to do their best work when they use their own calculator or one which the school has provided and which students have used in instructional settings prior to the test.

Policy

It is the policy of the State Department of Education, therefore, that students will be allowed to use any calculator provided to them or any calculator of their choosing, including scientific and graphics calculators, on the CAPT.

Additional Information

In implementing this policy, schools and school districts should be aware that:

- all four current National Science Foundation-funded secondary mathematics curriculum development projects integrate the graphics calculator into daily instruction and assessment;
- the College Board's new policy for the SATs also provides that, in an effort to assure true calculator equity, students will be allowed to use the calculator with which they are most familiar and comfortable; and
- nothing in this policy should be used to discourage the purchase and use of graphics calculators for ongoing mathematics instruction, and eventually by all students on all tests.

Calculators and the Education of Youth

Calculators are widely used at home and in the workplace. Increased use of calculators in school will ensure that students' experiences in mathematics will match the realities of everyday life, develop their reasoning skills, and promote the understanding and application of mathematics. The National Council of Teachers of Mathematics therefore recommends the integration of the calculator into the school mathematics program at all grade levels in classwork, homework and evaluation.

Instruction with calculators will extend the understanding of mathematics and will allow all students access to rich, problem-solving experiences. This instruction must develop students' ability to know how and when to use a calculator. Skill in estimation and the ability to decide if the solution to a problem is reasonable are essential adjuncts to the effective use of the calculator.

Evaluation must be in alignment with normal, everyday use of calculators in the classroom. Testing instruments that measure students' understanding of mathematics and its applications must include calculator use. As the availability of calculators increases and the technology improves, testing instruments and evaluation practices must be continually upgraded to reflect these changes.

The National Council of Teachers of Mathematics recommends that all students use calculators to:

- explore and experiment with mathematical ideas such as patterns, numerical and algebraic properties, and functions;
- develop and reinforce skills such as estimation, computation, graphing and analyzing data;
- focus on problem-solving processes rather than the computations associated with problems;
- perform the tedious computations that often develop when working with real data in problem situations;
- gain access to mathematical ideas and experiences that go beyond those levels limited by traditional paper-and-pencil computation.

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The National Council of Teachers of Mathematics also recommends that every mathematics teacher at every level promote the use of calculators to enhance mathematics instruction by:

- modeling the use of calculators in a variety of situations;
- using calculators in computation, problem-solving, concept development, pattern recognition, data analysis and graphing;
- incorporating the use of calculators in testing mathematical skills and concepts;
- keeping current with the state-of-the-art technology appropriate for the grade level being taught;
- exploring and developing new ways to use calculators to support instruction and assessment.

The National Council of Teachers of Mathematics further recommends that:

- school districts conduct staff development programs that enhance teachers' understanding of the use of appropriate state-of-the-art calculators in the classroom;
- teacher preparation institutions develop preservice and in-service programs that use a variety of calculators, including graphing calculators, at all levels of the curriculum;
- educators responsible for selecting curriculum materials make choices that reflect and support the use of calculators in the classroom;
- publishers, authors, and test and competition writers integrate the use of calculators at all levels of mathematics;
- mathematics educators inform students, parents, administrators, and school boards about the research that shows the advantages of including calculators as an everyday tool for the student of mathematics.

Research and experience have clearly demonstrated the potential of calculators to enhance students' learning in mathematics. The cognitive gain in number sense, conceptual development, and visualization can empower and motivate students to engage in true mathematical problem solving at a level previously denied to all but the most talented. The calculator is an essential tool for all students of mathematics.

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CONCEPTS TO MANIPULATIVES

THE FOLLOWING IS A LISTING OF <u>SOME</u> OF THE MANIPULATIVES THAT CAN EFFECTIVELY BE USED TO TEACH THE GIVEN CONCEPT.

| Concepts | Manipulatives |
|--|---|
| Angles | protractors, compasses, geoboards, miras, rulers, tangrams, pattern blocks |
| Area | geoboards, color tiles, base-ten blocks, decimal squares, cubes, tangrams, pattern blocks, rulers, fraction models |
| Classification, sorting | attribute blocks, cubes, pattern blocks, tangrams, 2-color counters, Cuisenaire rods, dominoes, geometric solids, money, numeral cards, base-ten materials, polyhedra models, geoboards, decimal squares, fraction models |
| Coordinate geometry | geoboards |
| Constructions | compasses, protractors, rulers, miras |
| Counting | cubes, 2-color counters, color tiles, Cuisenaire rods, dominoes, numeral cards, spinners, 10-frames, number cubes, money, calculators |
| Decimals | decimal squares, base-ten blocks, money, calculators, number cubes, numeral cards, spinners |
| Equations/inequalities Equality/inequality Equivalence | algebra tiles, math balance, calculators, 10-frames, balance scale, color tiles, dominoes, money, numeral cards, 2-color counters, cubes, Cuisenaire rods, decimal squares, fraction models |
| Estimation | color tiles, geoboards, balance scale, capacity containers, rulers, Cuisenaire rods, calculators |
| Factoring | algebra tiles |
| Fact strategies | 10-frames, 2-color counters, dominoes, cubes, numeral cards, spinners, number cubes, money, math balance, calculators |
| Fractions | fraction models, pattern blocks, base-ten materials, geoboards, clocks, color tiles, cubes, Cuisenaire rods, money, tangrams, calculators, number cubes, spinners, 2-color counters, decimal squares, numeral cards |
| Integers | 2-color counters, algebra tiles, thermometers, color tiles |
| | |

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| Concepts | Manipulatives | | |
|--|---|--|--|
| Logical reasoning | attribute blocks, Cuisenaire rods, dominoes, pattern blocks, tangrams, number cubes, spinners, geoboards | | |
| Measurement | balance scale, math balance, rulers, capacity containers, thermometers, clocks, geometric solids, base-ten materials, color tiles | | |
| Mental math | 10-frames, dominoes, number cubes, spinners | | |
| Money | money | | |
| Number concepts cubes, 2-color counters, spinners, number cubes, calcul dominoes, numeral cards, base-ten materials, Cuisenair fraction models, decimal squares, color tiles, 10-frames, | | | |
| Odd, even, Prime, composite | color tiles, cubes, Cuisenaire rods, numeral cards, 2-color counters | | |
| Patterns | pattern blocks, attribute blocks, tangrams, calculators, cubes, color tiles, Cuisenaire rods, dominoes, numeral cards, 10-frames | | |
| Percent | base-ten materials, decimal squares, color tiles, cubes, geoboards, fraction models | | |
| Perimeter/ Circumference | geoboards, color tiles, tangrams, pattern blocks, rulers, base-ten materials, cubes, fraction circles, decimal squares | | |
| Place value | base-ten materials, decimal squares, 10-frames, Cuisenaire rods, math balance, cubes, 2-color counters | | |
| Polynomials | algebra tiles, base-ten materials | | |
| Probability | spinners, number cubes, fraction models, money, color tiles, cubes, 2-color counters | | |
| Pythagorean Theorem | geoboards | | |
| Ratio/proportion | color tiles, cubes, Cuisenaire rods, tangrams, pattern blocks, 2-color counters | | |
| Similarity/congruence | geoboards, attribute blocks, pattern blocks, tangrams, miras | | |
| Size/shape/color | attribute blocks, cubes, color tiles, geoboards, geometric solids, pattern blocks, tangrams, polyhedra models | | |
| Spatial visualization | tangrams, pattern blocks, geoboards, geometric solids, polyhedra models, cubes, color tiles | | |

| Concepts | Manipulatives | |
|--|--|--|
| Square/cubic numbers | color tiles, cubes, base-ten materials, geoboards | |
| Surface area | color tiles, cubes | |
| Symmetry | geoboards, pattern blocks, tangrams, miras, cubes, attribute blocks | |
| Tessellations | pattern blocks, attribute blocks | |
| Transformational geom.: translations, rotations, reflections | geoboards, cubes, miras, pattern blocks, tangrams | |
| Volume | capacity containers, cubes, geometric solids, rulers | |
| Whole numbers | base-ten materials, balance scale, number cubes, spinners, color tiles, cubes, math balance, money, numeral cards, dominoes, rulers, calculators, 10-frames, Cuisenaire rods, clocks, 2-color counters | |

MANIPULATIVES TO CONCEPTS

THE FOLLOWING IS A LISTING OF <u>SOME</u> OF THE CONCEPTS THAT CAN EFFECTIVELY BE TAUGHT USING THE GIVEN MANIPULATIVES.

| Manipulative | Concepts | | | |
|---------------------|--|--|--|--|
| Algebra tiles | integers, equations, inequalities, polynomials, similar terms, factoring, estimation | | | |
| Attribute blocks | sorting, classification, investigation of size, shape, color, logical reasoning, sequencing, patterns, symmetry, similarity, congruence, thinking skills, geometry, organization of data | | | |
| Balance scale | weight, mass, equality, inequality, equations, operations on whole numbers, estimation, measurement | | | |
| Base-ten blocks | place value, operations on whole numbers, decimals, decimal- fractional-percent equivalencies, comparing, ordering, classification, sorting, number concepts, square & cubic numbers, area, perimeter, metric measurement, polynomials | | | |
| Calculators | problems with large numbers, problem solving, interdisciplinary problems, real-life problems, patterns, counting, number concepts, estimation, equality, inequality, fact strategies, operations on whole numbers, decimals, fractions | | | |
| Capacity containers | measurement, capacity, volume, estimation | | | |
| Clocks | time, multiplication, fractions, modular arithmetic, measurement | | | |
| Color tiles | color, shape, patterns, estimation, counting, number concepts, equality, inequality, operations on whole numbers & fractions, probability, measurement, area, perimeter, surface area, even & odd numbers, prime & composite numbers, ratio, proportion, percent, integers, square & cubic numbers, spatial visualization | | | |
| Compasses | constructions, angle measurement | | | |
| Cubes | number concepts, counting, place value, fact strategies – especially turnaround facts, classification, sorting, colors, patterns, square & cubic numbers, equality, inequalities, averages, ratio, proportion, percent, symmetry, spatial visualization, area, perimeter, volume, surface area, transformational geometry, operations on whole numbers & fractions, even & odd numbers, prime & composite numbers, probability | | | |

| Concepts |
|--|
| classification, sorting, ordering, counting, number concepts, comparisons, fractions, ratio, proportion, place value, patterns, even & odd numbers, prime & composite numbers, logical reasoning, estimation, operations on whole numbers |
| decimals - place value, comparing, ordering, operations, classification, sorting, number concepts, equality, inequality, percent, perimeter, area |
| counting, number concepts, fact strategies, classification, sorting, patterns, logical reasoning, equality, inequality, mental math, operations on whole numbers |
| fractions - meaning, recognition, classification, sorting, comparing, ordering, number concepts, equivalence, operations, perimeter, area, percent, probability |
| size, shape, counting, area, perimeter, circumference, symmetry, fractions, coordinate geometry, slopes, angles, Pythagorean Theorem, estimation, percent, similarity, congruence, rotations, reflections, translations, classification, sorting, square numbers, polygons, spatial visualization, logical reasoning |
| shape, size, relationships between area & volume, volume, classification, sorting, measurement, spatial visualization |
| equality, inequality, operations on whole numbers, open sentences, equations, place value, fact strategies, measurement, logical reasoning |
| symmetry, similarity, congruence, reflections, rotations, translations, angles, parallel & perpendicular lines, constructions |
| money, change, comparisons, counting, classification, sorting, equality, inequality, operations on whole numbers, decimals, fractions, probability, fact strategies, number concepts |
| counting, number concepts, fact strategies, mental math, operations on whole numbers, fractions, decimals, probability, generation of problems, logical reasoning |
| counting, classification, sorting, comparisons, equality, inequality, order, fact strategies, number concepts, operations on whole numbers, fractions, decimals, logical reasoning, patterns, odd & even numbers, prime & composite numbers |
| |

| Manipulative | Concepts |
|-------------------------|---|
| Pattern blocks | patterns, one-to-one correspondence, sorting, classification, size, shape, color, geometric relationships, symmetry, similarity, congruence, area, perimeter, reflections, rotations, translations, problem solving, logical reasoning, fractions, spatial visualization, tessellations, angles, ratio, proportions |
| Polyhedra models | shape, size, classification, sorting, polyhedra, spatial visualization |
| Protractors | constructions, angle measurement |
| Rulers Tape measures | measurement, area, perimeter, constructions, estimation, operations on whole numbers, volume |
| Spinners | counting, number concepts, operations on whole numbers, decimals, fractions, fact strategies, mental math, logical reasoning, probability, generation of problems |
| Tangrams | geometric concepts, spatial visualization, logical reasoning, fractions, similarity, congruence, area, perimeter, ratio, proportion, angles, classification, sorting, patterns, symmetry, reflections, translations, rotations |
| Ten-frames | fact strategies, mental math, number concepts, counting, equality, inequality, place value, patterns, operations on whole numbers |
| Thermometers | temperature, integers, measurement |
| Two-color counters | counting, comparing, sorting, classification, number concepts, fact strategies, even & odd numbers, equality, inequality, operations, ratio, proportions, probability, integers |

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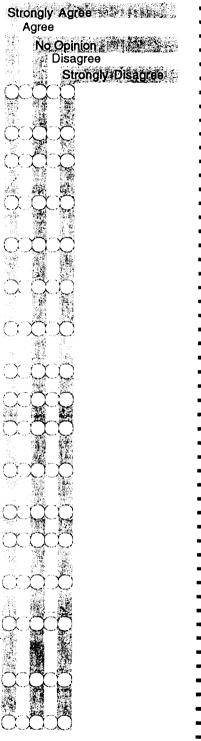
Mathematics Teacher Questionnaire

Instructions: You may use either a #2 pencil or blue or black ball point pen to complete the questionnaire. Please do not use a felt tip pen.

A. Teacher Opinions

Questions 1 - 26. Please give your opinion about each of the following statements. Darken one circle on each line.

- My principal is supportive of different approaches to teaching mathematics.
- 2. The more mathematics "drill" problems students work on in a class period, the more they will learn.
- 3. Virtually all children can learn to think mathematically.
- Most students learn best when grouped with students of similar abilities.
- 5. It is important for most students to learn to perform complex computations with speed and accuracy.
- Most parents fear that calculator use will inhibit learning basic number skills.
- 7. Mathematics for all students requires eliminating tracking by ability.
- 8. Most teachers in this school regularly share ideas and materials related to mathematics instruction.
- 9. Students need to master computation before going on to algebra.
- 10. The testing program in my state/district dictates what I teach.
- 11. Students should be able to use calculators anytime, other than when practicing basic calculations.
- 12. Including real-life applications disrupts the flow of mathematics instruction.
- 13. I organize my curriculum around the textbook.
- Manipulatives or "hands-on" materials help many students understand mathematics
- Most teachers in this school contribute actively to making decisions about the mathematics curriculum.
- 16. Most mathematics teachers in this school regularly observe each other teaching classes as part of sharing and improving instructional strategies.
- 17. The primary purpose of mathematics instruction is to prepare students for further study in mathematics.



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- I feel well-prepared to phrase questions to encourage more openended investigations.
- 19. I feel well-prepared to use the textbook as a resource rather than as the primary instructional tool.
- I feel well-prepared to manage a class of students who are using manipulatives.
- 21. I feel well-prepared to teach heterogeneous groups.
- 22. I feel well-prepared to use cooperative learning groups in mathematics instruction.
- 23. I feel well-prepared to use calculators as an integral part of mathematics instruction.
- I feel well-prepared to use computers as an integral part of mathematics instruction.
- 25. I feel well-prepared to use a variety of alternative assessment strategies.
- I feel well-prepared to involve parents in the mathematics education of their children.

B. Experience with NCTM Standards

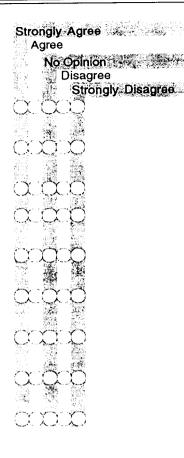
Not sure. (Skip to question 36.)

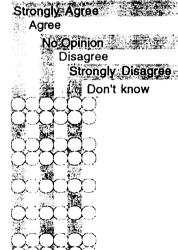
27. Are you aware that the National Council of Teachers of Mathematics has prepared Curriculum and Evaluation Standards, generally called the NCTM "Standards," for mathematics instruction? Darken one circle.

| () | Yes, I am well aware of the Standards. (Go to question 28.) |
|-----|---|
| Õ. | Yes, I have heard of the Standards, but I don't know very much about them at this |
| | time. (Skip to question 36.) |
| () | No Lam not aware of the Standards. (Skip to Question 36.) |

Questions 28 - 35. Indicate the extent to which you agree with each of the following statements. Darken one circle on each line.

- 28. I am well informed about the Standards for the grades I teach.
- 29. I am prepared to explain the Standards to my colleagues.
- If the Standards are followed, "paper and pencil" activities will disappear.
- 31. The Standards are just another fad, like new math.
- Our district has made changes in the mathematics curriculum based on the Standards.
- Most of the mathematics teachers in my school are well informed about the Standards.
- Mathematics teachers in my school have changed what and how they teach based on the Standards.
- 35. Our district is organizing staff development based on the Standards.





| S | are you aware that the National Council of Teachers of Mathematics ha Standards for Teaching Mathematics, generally called the NCTM "Teach Darken one circle. | |
|--------------------|---|---|
| | Yes, I am well aware of the Teaching Standards. (Go to Question Yes, I have heard about the Teaching Standards, but I don't know this time. (Skip to Section C.) No, I am not aware of the Teaching Standards. (Skip to Section C Not sure. (Skip to section C.) | very much about them at |
| Ques | tions 37 - 43. Indicate the extent to which you agree with each of | Strongly Agree |
| the f | ollowing statements. Darken one circle on each line. | No Opinion Disagree Strongly Disagree Don't know |
| 38. 39. | I am well informed about the Teaching Standards for my grade level. I am prepared to explain the Teaching Standards to my colleagues. The kind of mathematics teaching portrayed in the Teaching Standards is not possible with real children. The mathematics teachers in my school are generally well | |
| 41. | informed about the Teaching Standards. Most mathematics teachers in my school have changed how they teach based on the Teaching Standards. | |
| 42. | Our district has changed how it conducts in-service based on the | |
| 43. | Teaching Standards. Our district has changed how it evaluates teachers based on the Teaching Standards. | |
| c. c | Classroom Practices | Configuration Configuration |
| class t questic | ions 44-49. Throughout this section, you are asked to think about a phat you teach. If you teach more than one mathematics class per day, pons for the <u>first</u> mathematics class you teach each day. The street of this class? | particular mathematics please answer these |
| 45. W | Which of the following best describes the ability makeup of this class | 6? Darken one circle. |
| | Primarily high ability students Primarily low ability students Primarily average ability students Students of widely differing ability levels | |
| 46. | Instructional Emphases: Think about your plans for this mather course. How much emphasis will each of the following topics receive first class of the day is in grades K-4, 46B if grades 5-12, then go to | e? Answer 46A if your |
| 46A. | | Heavy nphasis |
| | a. Estimation b. Number sense and numeration | <u>g</u> |
| | c. Concepts of whole number operations | 8 |
| | d. Whole number computation e. Geometry and spatial sense | 2 |
| | f. Measurement | <u>ර</u> |
| | g. Statistics and probability h. Fractions and decimals i. Patterns and relationships | 일 음 |
| | Go to Question 47. | • |

| 47. C | a. Number and number relationshipsb. Number system and number theoryc. Computation | Emphasis | Emphasis |
|--|--|--|--|
| | d. Patterns e. Functions f. Algebra g. Statistics h. Probability i. Measurement j. Geometry from a synthetic perspective k. Geometry from an algebraic perspective I. Trigonometry m. Discrete mathematics n. Conceptual underpinning of calculus o. Mathematical structure | | |
| | Class Activities: About how often do students in to activities? Darken one circle on each line | | Almost every day. Several times a week About once a week Less than once a week Never |
| a. b. c. d. e. f. g. h. i. | Do mathematics problems on worksheets Work in small cooperative groups Work in class on mathematics projects Listen and take notes during presentation by tea Make conjectures and explore possible methods mathematical problem Learn about mathematics through real-life app Write their reasoning about how to solve a prob Use calculators Use computers | to solve a lications | |
| | How are the materials for manipulative or "hands-on Darken all circles that apply. As a part of a published basal textbook program. | 1 | |
| | As a part of a published non-textbook program As a part of an unpublished program developed Some or all are provided by my school or distri I provide some or all of the materials myself an This class does not use hands-on materials | I by my scho ict, but I plan | ol or district their use on my own |
| | How are calculators provided for use in this mathe | ematics class | . Darken all circles |
| | The school or district provides calculators for a The school or district provides calculators for so Some students provide their own calculators for Calculators are provided by another source Some students in this class don't use calculators | ome students r use in this _(Specify:_ | to use on a rotating basis class |

| D. | Prof | fessional Development | | | | |
|-----|--|--|---------------|--------------------|------------------|------|
| 50. | During the <i>last twelve months</i> , what is the <i>total</i> amount of time you have spent on professional development in mathematics or the teaching of mathematics? Include attendance at professional meetings and conferences, workshops, and courses. Darken one circle . | | | | | |
| | | None Less than 6 hours 6 - 15 hours 16 - 35 hours More than 35 hours | | | | |
| 51. | | en was your most recent course or in nematics teaching? Darken one | | cation experien | ce in mathematic | s or |
| | | Within the last 3 months 3 - 6 months ago 7 - 12 months ago 1 - 3 years ago More than 3 years ago | | | | |
| 52. | How | pose you wanted to find out more above likely would you be to use each of the circle on each line. | | | | |
| | one | circle on each line, | Not Likely | Somewhat Likely | Very Likely | |
| | c. d. e. f. g. | Other teachers in my school Teachers outside my school Principal District personnel State department personnel University-based personnel Meetings of professional organizatio In-service workshops Journals Publishers and sales representative | 9 | | | |
| E. | Tea | cher Background | | | | |
| 53. | Indic | ate your gender. Darken one ci | rcle. | | | |
| | (2) | Male Female | | | | |
| 54. | Whic | ch best describes you? Darken o | ne circle. | | | |
| | | American Indian or Alaskan Native African-American Asian or Pacific Islander Hispanic, regardless of race White (not of Hispanic origin) Prefer not to answer | | | | |
| | 8 | White (not of Hispanic origin) | | | | |

| 55. | How many years have you taught at either the elementary or secondary level prior to this school year? Darken one circle. |
|-----|---|
| | Fewer than three years 3 - 5 years 6 - 10 years 11 - 15 years 16 - 20 years More than 20 years |
| 56. | What grade levels do you currently teach? Darken all circles that apply. |
| | O O O O O O O O O O O O O O O O O O O |
| 57. | What is the total number of mathematics classes you teach each day? Darken one circle. |
| | 00000 |
| | 1 2 3 4 5 6 |
| 58. | Do you have a major or minor in <u>mathematics</u> at the undergraduate or graduate level? Darken one circle. |
| | ○ Yes ○ No |
| 59. | Do you have a major or minor in <u>mathematics education</u> at the undergraduate or graduate level? Darken one circle . |
| | ○ Yes ○ No |
| | <u> </u> |
| | |
| | |

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Components:

Physical Environment
Language
Teaching Methodology
Behavior Management
Academic Evaluation
Classroom Integration

Introduction:

The following are Standards and guidelines for an equitable mathematics and science teacher and classroom. Each section defines the category and describes factors for promoting maximum student learning in mathmatics and science. Classroom characteristics and teacher behaviors included in "Profile of an Equitable Math and Science Classroom and Teacher" reflect current research of effective teaching and learning, as well as a focus on promoting equity.

It is important to remember that the teacher and a classroom do not exist in isolation, but are part of a larger context that includes school and district policies and practices, administrative support of equity, other teachers, peer influences on students and parental involvement. Therefore, this deals with only part of the total picture.

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1. Physical Environment

Definition: The physical environment of the classroom includes:

- · displays on bulletin boards,
- · posters and presentations used to decorate the room,
- · greetings and messages posted on walls, and
- · configuration of desks and arrangement of room.

- A. Wall displays show both male and female representatives of various races, cultures and physical disabilities actively engaged in science and mathematics activities -- from historical to present.
- B. Wall displays invite and support interest in science and mathematics of all students.
- C. Seating arrangements and placement of furniture are flexible and facilitate the integration of all members of the class into learning activities.
- D. Classroom environment encourages movement of the teacher to be close to all students.
- E. Classroom environment is barrier free.
- F. The teacher extends student learning beyond the walls of the classroom into the community through partnerships with businesses, parents and community groups.

2. Curriculum

Definition: The curriculum of a classroom includes:

- · formal and informal content taught through lessons,
- · all activities related to lessons, and
- · all aspects of the teacher's program.

- A. Math and science activities and lessons are multicultural. They identify contributions of various cultures to math and science, and present various cultural perspectives about math and science topics.
- B. Information is presented by the teacher using a variety of methods that appeal to all students and invite the participation of under-represented students.
- C. The teacher organizes math and science instruction to insure that students learn to cooperate with students who are different.
- D. The teacher encourages and enables students to examine science and mathematics from a variety of cultural perspectives.
- E. The teacher carefully selects textbooks and resource materials from an equity perspective. If biases exist, the teacher discusses them with students, including how biased materials can effect learning.
- F. The teacher has an extensive background in mathematics and/or science content.
- G. The teacher is very secure with teaching mathematics and science content and can creatively utilize various methods of content presentation.
- H. The teacher designs rigorous mathematics and/or science lessons that challenge all students.
- 1. The teacher involves parents in the mathematics and/or science learning of students.
- J. The teacher includes learning activities that will develop skills, such as spatial skills, that have disparate development in students.

3. Language

Definition: The language of the classroom includes:

- · the language and style of language used by the teacher, and
- the language and style of language the teacher allows students to use.

- A. The teacher uses inclusionary terms for people in all written and oral communication. Inclusionary terms DO NOT assign positions to a specific gender, race or ethnicity (i.e. fireman vs. firefighter or "the student, he" vs. "the students, they" or "the student, he or she")
- B. The teacher consciously uses oral or written examples of women involved in science and mathematics activities.
- C. The teacher works with students to develop inclusionary language and encourages its use.
- D. The teacher does not allow any verbal harassment of one student by another.
- E. The teacher discusses the negative impact of derogatory terms in reference to race, gender, ethnicity, physical disability and sexual preference, and how this effects the learning environment.
- F. The teacher integrates current linguistic use into his/her language patterns, such as changing "handicapped" to "disabled" and changing "Black heritage" to "African American heritage".

4. Teaching Methodology

Definition: The teaching methodology in the classroom includes:

- · style of presentation,
- · time devoted to presentation, and
- · method of attention directed at students.

- A. The teacher provides the same amount of teaching attention to all students.
- B. The teacher varies the type of teaching attention to meet students' needs and learning styles.
- C. The teacher ensures the equal participation of all students in classroom discussions through various methods.
- D. The teacher uses a variety of presentation styles during mathematics and science lessons to keep all students engaged and involved in learning.
- E. The teacher analyzes interactions with students for differential patterns and takes action to counteract and balance differences.
- F. The teacher is knowledgeable of various methods for presenting content.
- G. The teacher keeps updated on new teaching methods through staff development and reading.
- H. The teacher collaborates and discusses teaching methods with colleagues.
- 1. The teacher utilizes inquiry as a mode for student learning.
- J. The teacher uses math manipulatives and/or science experiments to promote learning by all students.
- K. The teacher actively engages all students in discussion in science and math lessons, paying careful attention to the involvement of less verbal, aggressive students.
- L. The teacher uses small groups to promote the verbal participation of all in discussions of math and science.
- M. The teacher encourages students to take more math and science courses, and to get involved in informal math and science learning activities, such as events at science museums, etc.
- N. The teacher prevents passive non-participation of students by engaging all students in discussions.
- O. The teacher makes sure that all students set up and use science and math equipment.

Behavior Management

Definition: The behavior management of the classroom includes:

- style the teacher uses to control student behavior,
- · time the teacher takes to control student behavior, and
- methods used by the teacher to control student behavior.

- A. The teacher explicitly informs students in advance of acceptable and unacceptable behavior in the science and mathematics classroom.
- B. The teacher explicitly informs students in advance of the consequences of behavior.
- C. The teacher regularly praises students equally for good behavior.
- D. The teacher is consistent when applying behavior management techniques.
- E. The teacher DOES NOT allow any student to harass another student in any way.

6. Academic Evaluation

Definition: The academic evaluation of the classroom includes:

- style and systems used by the teacher to assess student performance,
- style and systems used by the teacher to evaluate student performance, and
- style and systems used by the teacher to report student academic performance.

- A. The teacher has high academic expectations for all students and expectations for students are not influenced by students' race, gender, ethnicity or physical disability.
- B. The teacher has analyzed personal biases that may influence expectations for students.
- C. The teacher communicates high academic expectations to all students.
- D. The teacher praises students for the intellectual quality of their math and science work, irrespective of the student's race, gender, national origin or physical disability from a set of criteria which has been announced to the students.
- E. The teacher uses a variety of methods of authentic assessment to evaluate student performance.
- F. The teacher experiments with different methods of authentic assessment, keeps track of the effective methods and continuously tries to improve classroom assessment.

7. Classroom Integration

Definition: Classroom integration includes:

- structure used to facilitate student social and academic cooperation, and
- activities used to facilitate student social and academic cooperation.

Ideal:

- A. The teacher utilizes a variety of learning activities in math and science that will help students to learn from one another and work together effectively.
- B. The teacher integrates students of all learning styles and abilities into heterogeneous groups.
- C. The teacher structures math and science activities to promote the development of leadership skills among all students.
- D. The teacher encourages students to identify and analyze their participation and involvement in groups, and to develop strategies for increasing effectiveness.

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